MATH 115-001 **Name**________________________________

Test #1 - Spring 2004

Directions Work on this test without the aid of any other person, and provide no aid to any other person. You may refer to your notes, textbook, internet sources, or any other books you care to consult. You may use computer/calculator tools of your choice. Attach this cover sheet to your solutions and submit it with your completed test on later than 10:00 AM on Monday, April 5.

Work these problems in the spirit of the approach to problem solving (modeling) used in this course. (See the attached guidelines on "Fitting a Model to Data.".) Pay particular attention to clearly communicating your thoughts, problem solving processes, and conclusions.

Sign one of the following two statements.

Statement: I worked this test in compliance with the above directions.

(Signature)

Statement: I am unable to sign the statement above due to the exception(s) listed below.

(Signature)

Problem 1. The table below shows the world record for the one mile run in minutes and seconds between 1913 and 1985. (Actually, some data has been omitted for the sake of brevity.) Explain and show how to develop graphical and functional models for the relationship between the world record time for the one mile run and the number of years since 1910. It is suggested that you relate the record time in seconds to the number of years since 1910 (Why?). Carefully define any symbols you introduce, and discuss how well your model seems to fit the data. (Use either average error or sum of absolute errors.)

Year	1913	1915	1923	1931	1933	1934	1937	1942	1943	1944	1945
Time	4:14.4	4:12.6	4:10.4	4:09.2	4:07.6	4:06.8	4:06.4	4:04.6	4:02.6	4:01.6	4:01.4
Year	1954	1957	1958	1962	1964	1965	1966	1967	1975	1979	1985
Time	3:58.0	3:57.2	3:54.5	3:54.4	3:54.1	3:53.6	3:51.3	3:51.1	3:49.4	3:49.0	3:46.3

It looks like athletes continue to run the mile faster and faster as the years go by, but it seems impossible that eventually someone will run a mile in one minute. Use your graphical and functional models to estimate when it is likely that the mile could be run in 3 minutes 40 seconds (3:40.0). Look up the evolution, if any, of the world record for the mile since 1985 and comment on how well your model predicted the evolution of the world record.

Problem 2. The table below shows British Highway Code recommended safe stopping distances for cars. Use the data in the table to help you find graphical and functional mathematical models relating the various distances and the speed of the vehicle in mph.

Speed (mph)	Reaction Distance (ft)	Braking Distance (ft)	Total Stopping Distance (ft)
20	20	20	40
30	30	45	75
40	40	80	120
50	50	125	175
60	60	180	240
70	70	245	315
80			

Show how to use both your graphical and functional models to complete the table by filling in the appropriate distances for a car traveling at 80 mph.

Let R(s) = reaction distance for a car traveling s mph,
B(s) = braking distance for a car traveling s mph, and
T(s) = total stopping distance for a car traveling s mph.

Explain and show how to derive functional equations for R(s), B(s), and T(s).

Look up US recommended stopping distance data and comment on how those numbers compare to the British numbers.

Problem 3. The following table, based on data from the U.S. Department of Education, gives that total number of high school graduates, in thousands, in the indicated years since 1900.

Year	High School Grads (1000's)
1900	364
1910	502
1920	665
1930	746
1940	848
1950	1461
1960	2029
1970	2694
1980	3621
1990	4863

Determine an exponential function to fit the data for the number of high school graduates as a function of the number of years since 1900. Demonstrate that you have employed a numerical criterion to select your function from several candidates. Comment on how well your model fits the data. Use your function to predict the number of high school graduates in the year 2000 and in the year 2005. See if you can validate your estimates.

Modeling Exercises - Objectives

Approach modeling exercises with the intention to demonstrate accomplishment of the following objectives:

- ¬ Demonstrate understanding of the mathematical modeling process.
- ¬ Apply a mathematical modeling process to a variety of situations from the real world.
- ¬ Recognize that similar models may represent different situations.
- ¬ Recognize that several different models may represent a situation.
- ¬ Know the strengths and limitations of mathematical modeling as a method for solving real-world problems.
- ¬ Be adept in using some technological tools, such as calculators or computers, in problem solving.
- ¬ Use a variety of mathematical techniques in modeling and problem solving.
- ¬ In modeling from data:
    - o Model real phenomena with a variety of functions;
    - o Represents and analyze relationships using tables, rules, and graphs;
    - o Translate among tabular, symbolic, and graphical representations of functions;
    - o Analyze the effects of parameter changes on the graphs of functions;
    - o Use curve fitting to predict from data;
    - o Transform data to aid in data interpretation and prediction;
    - o Employ criteria for "goodness of fit."
- ¬ Skillfully communicate mathematical ideas, your thought process, and approach to problem solving in writing.
- ¬ Critically evaluate mathematical models and comment on their strengths and limitations.
- ¬ In general, demonstrate the following steps in fitting a model to data relative to a phenomenon.
    - o Formulate the key problem or question. (Show you understand the problem.)
    - o Communicate your preconceptions.
    - o Discuss the limitations, assumptions, and scope of your investigation.
    - o Collect or organize data.
    - o Analyze and interpret data.
    - o Choose and fit an appropriate model by varying parameters and testing goodness of fit by an identified numberical criterion (sum of errors, average error, percent error).
    - o Validate, summarize and report findings. (Reflect, describe, formulate, evaluate, support, generalize, and suggest.)

Below are listed some actions to avoid in the model development phases. However, those actions may be employed in the evaluation or critique of your models if you give reference credit to the device, software, or source:

- o The use of a calculator's or software application's pre-programmed curve fitting procedures.
- o Copying a formula or rule from a reference source.