http://my.nctm.org/standards/document/chapter6/reas.htm

What should be the teacher's role in developing reasoning and proof in grades 6 through 8?

Teachers in the middle grades can help students appreciate and use the power of mathematical reasoning by regularly engaging students in thinking and reasoning in the classroom. Fostering a mathematically thoughtful environment is vital to supporting the development of students' facility with mathematical reasoning.

The teacher plays an important role by creating or selecting tasks that are appropriate to the ages and interests of middle-grades students and that call for reasoning to investigate mathematical relationships. Tasks that require the generation and organization of data to make, validate, or refute a conjecture are often appropriate. For example, the examination of patterns associated with figurate numbers discussed above shows how a teacher can use the task both to stimulate students' investigation and to develop facility with mathematical reasoning and argumentation. Suitable tasks can arise in everyday life, although many will arise within mathematics itself. Teachers also serve as monitors of students' developing facility with reasoning. In order to use inductive reasoning appropriately, students need to know its limitations as well as its possibilities. Because many elementary and middle-grades tasks rely on inductive reasoning, teachers should be aware that students might develop an incorrect expectation that patterns always generalize in ways that would be expected on the basis of the regularities found in the first few terms. The following hypothetical example shows how a » teacher could help students develop a healthy appreciation for the power and limits of inductive reasoning.

A teacher asks students to determine how many segments of different lengths can be made by connecting pegs on a square geoboard that is 5 units on each side (a 5×5 square geoboard). Because the number of segments is large and some students will have difficulty being systematic in representing the segments on their geoboards, the teacher encourages the students to examine simpler cases to develop a systematic way to generate the different segments. The students approach this task by examining the number of segments on various subsquares on a 5×5 geoboard, looking at the growth from a 1×1 square to a 4×4 square, as shown in figure 6.34.

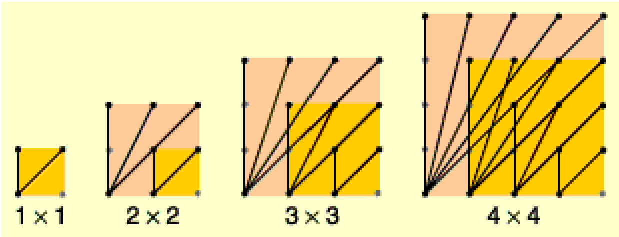

Fig. **6.34.** Segments of different lengths

The teacher helps the students see that each successive square contains the previous square within it. Thus, the number of segments on a 3×3 square can be found by adding the number of segments found on a 2×2 square to the number of new segments that can be created using the "new" pegs within the 3×3 square. The teacher has the students verify— by direct measurement or by treating the diagonal lengths as hypotenuses of right triangles—that all segments are really of different lengths. The students then record the number of segments of different lengths in each square and note the pattern of growth, as shown in figure 6.35.

Size of Square	Number of Segments of Different Lengths: Old + New	Total Number of Different Lengths
1 × 1	2	2
2 × 2	2 + 3	5
3 × 3	(2 + 3) + 4	9
4 × 4	(2 + 3 + 4) + 5	14
5 × 5	?	?

Fig. **6.35.** Students can record in a table like this one data about the number of segments of different lengths on a geoboard.

The teacher orchestrates a class discussion about the numbers in the table. Most students quickly detect a pattern of growth and are prepared to predict the answer for a 5 × 5 geoboard—20 different segments—because (2 + 3 + 4 + 5) + 6 = 14 + 6 = 20. In fact, many students are prepared to state a more general conjecture: The number of segments for an $N \times N$ square geoboard is the sum 2 + 3 + ... + (N + 1).

After the students have made their prediction, the teacher asks them to check its accuracy by actually making all the possible segments of different lengths on a 5 × 5 geoboard, as in figure 6.36. Because of their prior experience in systematically generating all possible segments, most of the students are able to find all the possibilities. In fact, most recognize that they need only to count the "new" segments and check to be sure that the segments are of different lengths from one another and from the segments already counted in the previous cases. The students note that there are twenty segments, as predicted, and most are content with the observation that all the new segments are of different lengths. But » some students discover that two segments—*AB* and *CD* in figure 6.36—are both five units long. Thus, there are only nineteen different lengths, rather than twenty as predicted. Most of the students are surprised at this result, although they recognize that it is correct.

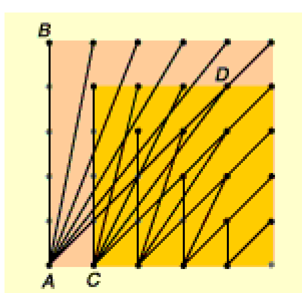

Fig. **6.36.** Line segments on a 5 × 5 geoboard

A teacher can use an example such as this as a powerful reminder that students should be cautious when generalizing inductively from a small number of cases, because not all patterns generalize in ways that we might wish or expect from early observations. This important lesson allows students to develop a healthy skepticism in their work with patterns and generalization.

Teachers need to monitor students' developing facility not only with inductive reasoning but also with deductive reasoning. In the middle grades, students begin to consider assertions such as the following: The diagonals of any given rectangle are equal in length. (See the "Geometry" section of this chapter for more discussion of how this assertion might be generated and verified by students.) An assertion such as this is tricky, at least in part because it is an implicitly conditional statement: If a shape is a rectangle, then its diagonals are equal in length. Thus, it is probably not surprising that some students will misapply this idea by inferring that any quadrilateral with diagonals of equal length must be a rectangle. Doing so reflects the erroneous view that if a statement is true then its converse is true. In this instance, the converse is not true because nonrectangular isosceles trapezoids also have diagonals of equal length, as do many other quadrilaterals.

Teachers in the middle grades need to be mindful of complexities in logical thinking and be alert in order to help students reason correctly. In this example, a teacher might have students use dynamic geometry software to investigate which types of quadrilaterals have diagonals of equal length. The software could allow students to see changes in the lengths of the diagonals instantly as they change the shape of the quadrilateral. A teacher might have students investigate quadrilaterals in general and particular types of quadrilaterals, including rectangles, squares, parallelograms, rhombuses, and trapezoids. The teacher might ask students to note which shapes have diagonals of equal length. If no one found such a shape, the teacher could ask them to construct an isosceles trapezoid with a given set of vertices, and the students would then see that this trapezoid has diagonals of equal length. This type of investigation can lead students to understand that even when a statement is true, its converse may be false.