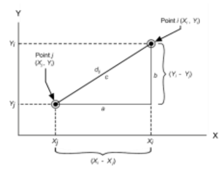
Spatial Statistics

GEOG 419: Lembo

Point Pattern Analysis

- Global methods to analyze point patterns across entire study region (or a map)
  - Quantitative tools for examining a spatial arrangement of point locations on the landscape
- Two common types of analysis
  - spacing of individual points – nearest neighbor analysis
    - Ex. fire stations locations – random or dispersed
      - Goal: equitable service throughout region
      - Design new configuration (e.g., relocating, new stations)
      - More or less dispersed than original configuration
  - nature of overall point pattern – are locations dispersed or clustered
    - Ex. diseased trees in a national forest
      - Widespread aerial spraying versus concentrated ground treatment

Center Point



Pythagorean theorem:

$a^2 + b^2 = c^2$

In a right-angled triangle, the square of the hypotenuse (c) is equal to the sum of the squares of the other two sides (a and b)

Euclidean (straight-line) distance:

$d_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$

FIGURE 4.2
Calculation of Euclidean Distance (d_{ij}) from Point i to Point j

TABLE 4.2

Worktable for Calculating Central Point

Distance matrix between points in figure 4.1

	A	B	C	D	E	F	G	Total Distance
A	0	2.59	1.93	1.68	1.55	2.56	2.90	13.21
B	2.59	0	1.96	3.33	3.82	3.86	3.31	18.87
C	1.93	1.96	0	1.58	2.34	1.92	1.41	11.14
D	1.68	3.33	1.58	0	0.91	0.89	1.58	9.97
E	1.55	3.82	2.34	0.91	0	1.58	2.47	12.67
F	2.56	3.86	1.92	0.89	1.58	0	1.14	11.95
G	2.90	3.31	1.41	1.58	2.47	1.14	0	12.81

Point "D" is the lowest total distance from all other points: 9.97

Euclidean distance from point A to point B:

$$d_{AB} = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}$$

$$= \sqrt{(2.8 - 1.6)^2 + (1.5 - 3.8)^2}$$

$$= 2.59$$

Locational coordinates: A (2.8, 1.5), B (1.6, 3.8)

Euclidean (straight-line) distance

- Total distance from all other points is lowest

Center Point

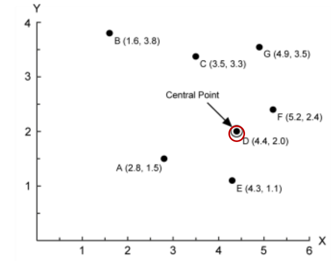


FIGURE 4.1
Graph of Locational Coordinates and Central Point

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Mean Center

- mean center – average location of a set of points
  - Center of gravity of point pattern (spatial distribution)
  - average X, Y values
  - equal weights

$$\overline{X}_c = \frac{\sum X_i}{n} \text{ and } \overline{Y}_c = \frac{\sum Y_i}{n}$$

where:

$\overline{X}_c$ = mean center of X

$\overline{Y}_c$ = mean center of Y

X_i = X coordinate of point i

Y_i = Y coordinate of point i

n = number of points in the distribution

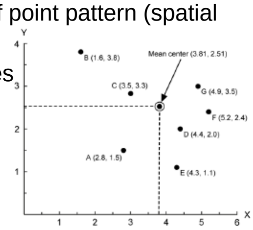

FIGURE 4.1
Graph of Locational Coordinates and Mean Center

Mean Center

- Outliers….
  - add point (15, 13)
  - Average location but…

TABLE 4.2

Worktable for Calculating Mean Center

Point	Locational coordinates *	
	X_i	Y_i
A	2.8	1.5
B	1.6	3.8
C	3.5	3.3
D	4.4	2.0
E	4.3	1.1
F	5.2	2.4
G	4.9	3.5

$n = 7$ $\sum X_i = 26.7$ $\sum Y_i = 17.6$

$\bar{X}_c = \frac{\sum X_i}{n} = \frac{26.7}{7} = 3.81$ $\bar{Y}_c = \frac{\sum Y_i}{n} = \frac{17.6}{7} = 2.51$

Mean center coordinates: (3.81, 2.51) *

* See figure 4.1 for graph of locational coordinates and mean center.

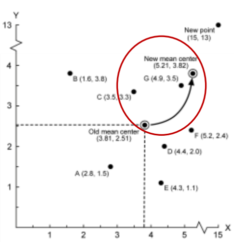

FIGURE 4.4
How an Outlier Might Affect Mean Center Location

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Mean Center

- geographic "center of population" – point where a rigid map of the country would balance if equal weights (i.e., location of each person) were situated over it

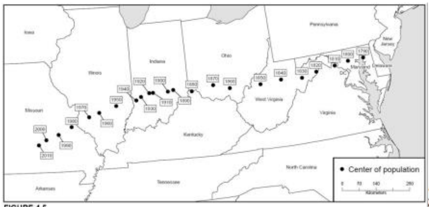

FIGURE 4.5
Geographic Center of U.S. Population, 1790 to 2010
Source: United States Bureau of the Census, 2010

Weighted mean center

- Unequal weights applied to points
  - Ex. retail store volume, city populations, etc.
  - Weights analogous to frequencies

$$\bar{X}_{wc} = \frac{\Sigma f_i X_i}{\Sigma f_i} \text{ and } \bar{Y}_{wc} = \frac{\Sigma f_i Y_i}{\Sigma f_i}$$

where $\bar{X}_{wc}$ = weighted mean center of X
$\bar{Y}_{wc}$ = weighted mean center of Y
f_i = frequency (weight) of point i

Weighted mean center

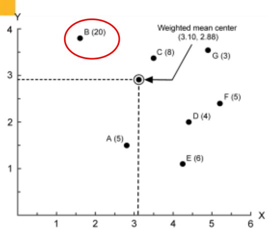

FIGURE 4.6
Graph of Point Locations, Weights (in Parentheses) and Weighted Mean Center

TABLE 4.4

Worktable for Calculating Weighted Mean Center

Point	Locational coordinates X_i	Locational coordinates Y_i	Weight f_i	Weighted coordinates $f_i X_i$	Weighted coordinates $f_i Y_i$
A	2.8	1.5	5	14.0	7.5
B	1.6	3.8	20	32.0	76.0
C	3.5	3.3	8	28.0	26.4
D	4.4	2.0	4	17.6	8.0
E	4.3	1.1	6	25.8	6.6
F	5.2	2.4	5	26.0	12.0
G	4.9	3.5	3	14.7	10.5

$n = 7$ $\quad \sum f_i = 51$ $\quad \sum f_i X_i = 158.1$ $\quad \sum f_i Y_i = 147.0$

$$\bar{X}_{wc} = \frac{\Sigma f_i X_i}{\Sigma f_i} = \frac{158.1}{51} = 3.10$$

$$\bar{Y}_{wc} = \frac{\Sigma f_i Y_i}{\Sigma f_i} = \frac{147.0}{51} = 2.88$$

Weighted mean center coordinates: (3.10, 2.88)

Spatial measures of dispersion

- standard distance – measures the amount of absolute dispersion in a point distribution
  - spatial equivalent to standard deviation
  - calculate Euclidean distance from each point to mean center

$$S_D = \sqrt{\frac{\Sigma (X_i - \overline{X}_c)^2 + \Sigma (Y_i - \overline{Y}_c)^2}{n}}$$

$$S_D = \sqrt{\left(\frac{\Sigma X_i^2}{n} - \overline{X}_c^2\right) + \left(\frac{\Sigma Y_i^2}{n} - \overline{Y}_c^2\right)}$$

Standard distance

TABLE 4.5

Worktable for Calculating Standard Distance

Point	X_i (Locational coordinates)	Y_i (Locational coordinates)	X_i^2	Y_i^2
A	2.8	1.5	7.84	2.25
B	1.6	3.8	2.56	14.44
C	3.5	3.3	12.25	10.89
D	4.4	2.0	19.36	4.00
E	4.3	1.1	18.49	1.21
F	5.2	2.4	27.04	5.76
G	4.9	3.5	24.01	12.25

$\overline{X}_c = 3.81$ $\overline{Y}_c = 2.51$ $\overline{X}_c^2 = 14.52$ $\overline{Y}_c^2 = 6.30$

$n = 7$ $\Sigma X_i^2 = 111.50$ $\Sigma Y_i^2 = 50.80$

$$S_D = \sqrt{\left(\frac{\Sigma X_i^2}{n} - \overline{X}_c^2\right) + \left(\frac{\Sigma Y_i^2}{n} - \overline{Y}_c^2\right)}$$

$$= \sqrt{\left(\frac{111.50}{7} - 14.52\right) + \left(\frac{50.80}{7} - 6.30\right)}$$

$$= 1.54$$

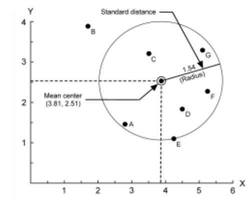

FIGURE 4.10
Graph of Point Locations, Mean Center, and Standard Distance

Relative Measure….

Weighted standard distance

- Used with weighted mean center

$$S_{WD} = \sqrt{\left(\frac{\Sigma f_i (X_i^2)}{\Sigma f_i} - \overline{X}_{wc}^2\right) + \left(\frac{\Sigma f_i (Y_i^2)}{\Sigma f_i} - \overline{Y}_{wc}^2\right)}$$

$$S_{WD} = \sqrt{\frac{\Sigma f_i (X_i - \overline{X}_{wc})^2 + \Sigma f_i (Y_i - \overline{Y}_{wc})^2}{\Sigma f_i}}$$

  - Difference
    - 1.54 vs. 1.70

TABLE 4.6

Worktable for Calculating Weighted Standard Distance

Point	f_i	X_i	X_i^2	$f_i(X_i)^2$	Y_i	Y_i^2	$f_i(Y_i)^2$
A	5	2.8	7.84	39.20	1.5	2.25	11.25
B	20	1.6	2.56	51.20	3.8	14.44	288.80
C	8	3.5	12.25	98.00	3.3	10.89	87.12
D	4	4.4	19.36	77.44	2.0	4.00	16.00
E	6	4.3	18.49	110.94	1.1	1.21	7.26
F	5	5.2	27.04	135.20	2.4	5.76	28.80
G	3	4.9	24.01	72.03	3.5	12.25	36.75

From earlier calculation of weighted mean center:

$\overline{X}_{wc} = 3.10$ $\overline{Y}_{wc} = 2.88$ $\overline{X}_{wc}^2 = 9.61$ $\overline{Y}_{wc}^2 = 8.29$

$\Sigma f_i = 51$ $\Sigma f_i(X_i)^2 = 584.01$ $\Sigma f_i(Y_i)^2 = 475.98$

$$S_{WD} = \sqrt{\left(\frac{\Sigma f_i(X_i)^2}{\Sigma f_i} - \overline{X}_{WC}^2\right) + \left(\frac{\Sigma f_i(Y_i)^2}{\Sigma f_i} - \overline{Y}_{WC}^2\right)}$$

$$= \sqrt{\left(\frac{584.01}{51} - 9.61\right) + \left(\frac{475.98}{51} - 8.29\right)}$$

$$= 1.70$$

Standard Deviational Ellipse

- Extends standard distance to include orientation of the point pattern
  - Calculated separately for X and Y
    - Average distance points vary from mean center on X and average distance points vary

$$SD_x = \sqrt{\frac{\Sigma\left(X_i - \bar{X}_c\right)^2}{n}} \text{ and } SD_y = \sqrt{\frac{\Sigma\left(Y_i - \bar{Y}_c\right)^2}{n}} \quad \textbf{(4.16)}$$

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Standard Deviational Ellipse

- cross of dispersion
- trigonometric function – angle of rotation
  - Rotated about mean center to minimize distance between both arms and points

TABLE 4.7

Summary Statistics for Standard Deviational Ellipse: Anti-Shipping Activity off the East African Coast

Year	# of incidents	$\bar{X}_c$	$\bar{Y}_c$	SD_x (miles)	SD_y (miles)	Rotation
2009	114	52.53	-1.00	379.7	604.7	45.4
2010	230	52.06	-1.58	415.9	956.3	56.3

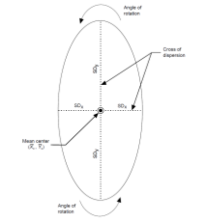
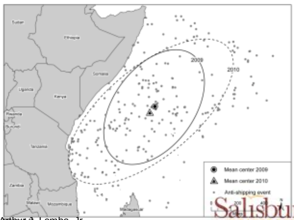


FIGURE 4.11
Standard Deviational Ellipse



Standard Deviational Ellipse: Anti-shipping Activity off the East African Coast, 2009 and 2010

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Nearest Neighbor Analysis – (NNA)

- Distance of each point to its nearest neighbor is measured and mean distance for all points is determined
  - Objective: describe the pattern of points in a study region and make inferences about the underlying process

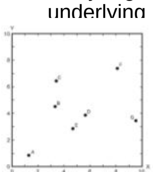

FIGURE 14.1
Location of Points for Nearest Neighbor Problem

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Nearest Neighbor analysis – (NNA)

- Compare calculated value from point data to theoretical point distributions
  - Outcomes: random, clustered, dispersed
  - average nearest neighbor distance is an absolute index
    - Dependent on distance measure (ex. miles, km, meters, etc.)
    - Minimum = 0 (clustered), maximum is function of point density
  - standardized nearest neighbor index (R) is often used
    - Comparison of data to random

$$R = \frac{\overline{NND}}{\overline{NND}_R} \qquad (14.5)$$

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Nearest Neighbor analysis – (NNA)

$$\overline{NND} = \frac{\sum NND}{n} \qquad (14.1)$$

where n = number of points.

$$\overline{NND}_R = \frac{1}{2\sqrt{Density}} \qquad (14.2)$$

where $\overline{NND}_R$ = mean nearest neighbor distance in a random pattern

$Density$ = number of points (n)/Area

$$\overline{NND}_R = \frac{1}{2\sqrt{.07}} = 1.89$$

$$R = \frac{\overline{NND}}{\overline{NND}_R} \qquad (14.5)$$

TABLE 14.1

Coordinates and Nearest Neighbor Information for Example*

Point	X	Y	NN	NND
A	1.3	0.9	E	3.94
B	3.2	4.4	C	2.00
C	3.3	6.4	B	2.00
D	5.6	3.8	E	1.36
E	4.8	2.7	D	1.36
F	8.1	7.4	G	4.21
G	9.4	3.4	D	3.82
SUM				18.69

where NN = nearest neighbor

NND = nearest neighbor distance

$$\overline{NND} = \frac{\sum NND}{n} = \frac{18.69}{7} = 2.67$$

* See figure 14.1 for graph of point locations.

$$R = \frac{2.67}{1.89} = 1.41$$

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NNA – R values

- Continuum…
  - Result?
  - Descriptive test.

$$R = \frac{2.67}{1.89} = 1.41$$

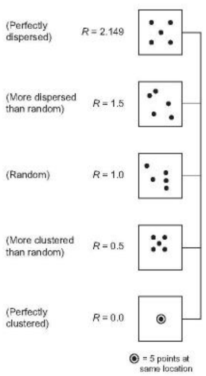
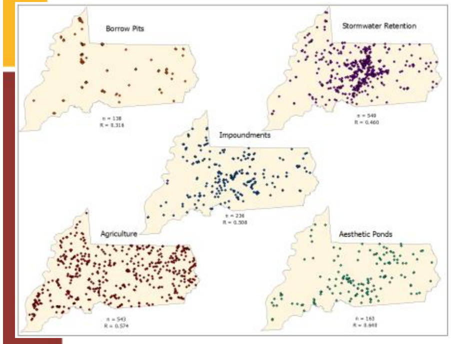


FIGURE 14.2
Continuum of R Values in Nearest Neighbor Analysis



Functional SWB

Results?

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Nearest neighbor analysis (nna)

- A difference test can be used to determine if the observed nearest neighbor index $(\overline{NNA})$ differs significantly from the theoretical norm $(\overline{NNA}_R)$
  - H_0: There is no difference between our distribution and a random distribution (Poisson)

$$Z_n = \frac{\overline{NND} - \overline{NND}_R}{\sigma_{\overline{NND}}} \quad (14.6)$$

where $\sigma_{\overline{NND}}$ = standard error of the mean nearest neighbor distances

The standard error for the nearest neighbor test can be estimated with the following formula:

$$\sigma_{\overline{NND}} = \frac{.26136}{\sqrt{n(Density)}} \quad (14.7)$$

where: n = number of points $Density$ = number of points (n)/Area

Nearest Neighbor Analysis

Primary Objective: Determine whether a random (Poisson) process has generated a point pattern

Requirements and Assumptions:

1. Random sample of points from a population
2. Sample points are independently selected

Hypotheses:

$H_0 : \overline{NND} = \overline{NND}_R$ (point pattern is random)

$H_A : \overline{NND} \neq \overline{NND}_R$ (point pattern is not random)

$H_A : \overline{NND} > \overline{NND}_R$ (point pattern is more dispersed than random)

$H_A : \overline{NND} < \overline{NND}_R$ (point pattern is more clustered than random)

Test Statistic:

$$Z_n = \frac{\overline{NND} - \overline{NND}_R}{\sigma_{\overline{NND}}}$$

Nearest neighbor analysis (nna) Example: Community Services in Toronto

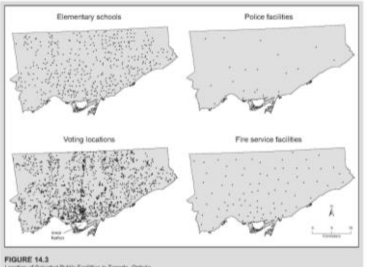

FIGURE 14.3
Location of Selected Public Facilities in Toronto, Ontario
Source: City of Toronto Planning Division

- Emergency services: fire and police
  - Seek dispersion to provide services equally
- Nonemergency services: polling sites and elementary schools
  - Seek clustering…why?

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Nearest neighbor analysis (nna) Example: Community Services in Toronto

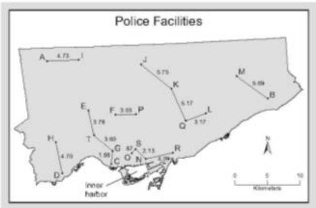


FIGURE 14.4
Nearest Neighbor Distances for Police Service Facilities in Toronto, Ontario

TABLE 14.2

Worktable for Nearest Neighbor Analysis: Police Service Facilities in Toronto, Ontario

$H_0: \overline{NND} = \overline{NND}_R$ (point pattern is random)
$H_A: \overline{NND} \neq \overline{NND}_R$ (point pattern is not random)

Calculate mean nearest neighbor distance:

$$\overline{NND} = \frac{\sum NND}{n} = \frac{72.60}{20} = 3.63$$

where: n = number of points

Calculate random nearest neighbor distance:

$$\overline{NND}_R = \frac{1}{2\sqrt{Density}}$$

where: Density = n/Area

$$\overline{NND}_R = \frac{1}{2\sqrt{\frac{20}{641.45}}} = \frac{1}{2\sqrt{0.0312}} = \frac{1}{0.1766} = 2.83$$

Calculate standardized nearest neighbor distance:

$$R = \frac{\overline{NND}}{\overline{NND}_R} = \frac{3.63}{2.83} = 1.28$$

Calculate test statistic:

$$Z_n = \frac{\overline{NND} - \overline{NND}_R}{\sigma_{\overline{NND}}}$$

where: Density = n/Area

$$\sigma_{\overline{NND}} = \frac{0.26136}{\sqrt{20(0.0312)}} = \frac{0.26136}{0.7899} = 0.3309$$

$$Z_n = \frac{3.63 - 2.83}{0.3309} = \frac{0.80}{0.3309} = 2.42 \quad (p = 0.0078)$$

- Result?

Nearest neighbor analysis (nna) Example: Community Services in Toronto

TABLE 14.3

Nearest Neighbor Values for Selected Public Facilities in Toronto, Ontario

Public Facility	$\overline{NND}$	Density	$\overline{NND}_R$	R	Z_n	p
Fire	2.06	.1278	1.40	1.47	8.21	.0000
Police	3.63	.0312	2.83	1.28	2.42	.0153
Elementary schools	0.59	.7358	0.58	1.02	0.95	.3439
Voting locations	0.27	2.4351	0.32	0.84	−12.23	.0000

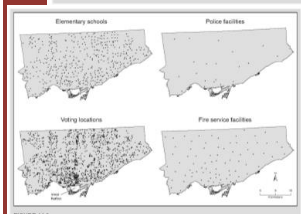


FIGURE 14.3
Location of Selected Public Facilities in Toronto, Ontario

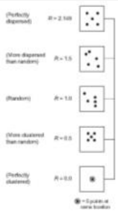


FIGURE 14.2
Continuum of R Values in Nearest Neighbor Analysis

Emergency services – more dispersed

Voting locations – more clustered

Elementary schools - random

Nearest neighbor analysis (nna) Example: Community Services in Toronto

- Issues to consider…
  - Study area boundaries – political boundary or research delimited
    - Doesn't impact NNA distances but does impact area (point density function)
  - Nearest feature – may be outside study area!
    - Problem with using political boundaries
  - More advanced techniques available – *Ripley's K*
    - Evaluates more than one nearest neighbor
    - Can define distances – How many police stations within 1km? 2km?

$$\sigma_{\overline{NND}} = \frac{.26136}{\sqrt{n(Density)}} \qquad \textbf{(14.7)}$$

where: n = number of points

General Issues in Inferential Spatial Statistics

- Geographers are interested in spatial patterns produced by physical or cultural processes
  - Explain patterns of points and areas
    - "global" overall arrangement
      - Random vs. Nonrandom spatial processes
    - "local" concentrations or absences
      - Clusters – points or areas within larger area
        - » Groups of high values – "hot spots"
        - » Groups of low values – "low spots"

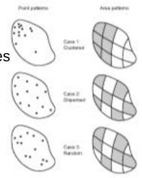


FIGURE 13.1
Types of Point and Area Patterns

Types of Spatial Patterns

- Compare existing pattern to theoretical pattern
- Clustered
  - Density of points varies significantly from one part of study area to another
    - Points: retail locations near highway interchange
    - Areas: registered majority political party affiliation
  - Patterns result from nonrandom factors
    - Accessibility, income, race, etc.

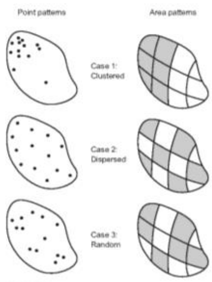


FIGURE 13.1
Types of Point and Area Patterns

Types of Spatial Patterns

- Dispersed
  - Uniformly distributed across study area
    - Suggests systematic spatial process
    - Area example: Central Place Theory – settlements are uniformly distributed across landscape to best serve needs of a dispersed rural population

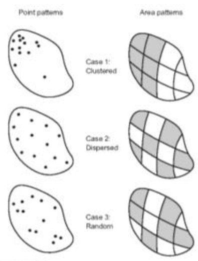


FIGURE 13.1
Types of Point and Area Patterns

Types of Spatial Patterns

- Random
  - No dominant trend toward clustering or dispersion
    - Suggests spatially random process (Poisson)
    - Ex. lightning strikes
- Geographic problems
  - Patterns typically appear as some combination of these three patterns
    - Along continuum…

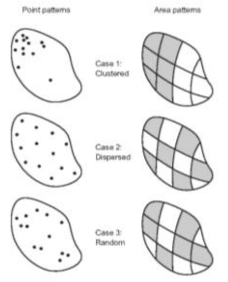


FIGURE 13.1
Types of Point and Area Patterns

Spatial Autocorrelation

- Tobler's Law – "Everything is related to everything else but near things are more related than distant things"
- spatial autocorrelation: measures the degree to which a geographic variable is correlated with itself through space
  - Positive, negative or non-existent
    - Positive spatial autocorrelation: objects near one another tend to be similar
      - Features with high values are near other features with high values, features with medium values are near other features with medium values, etc.
    - Negative spatial autocorrelation: objects near one another tend to have sharply contrasting values
      - Features with high values near features with low values
- Most geographic phenomena exhibit positive spatial autocorrelation
  - Examples: rainfall amounts, home values, etc.

Variogram

- Visualization of spatial autocorrelation
- variogram: scatterplot that display the differences in values between geographic locations against the differences in distances between the geographic locations
  - Y-axis: average *variance* (really half the variance) in *values* for a set of geographic objects
  - X-axis: *distance* between objects
  - Use plot to determine average difference in values at specific distances
    - Ex. 100 miles, 500 miles

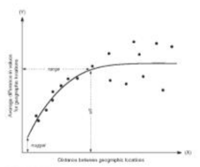


FIGURE 13.2
An Example of a Variogram

Geographic locations near one another tend to have smaller differences than geographic locations at greater distances (positive autocorrelation)!

Variogram

- Displayed as best-fitting curve (function)
  - Differences in values with distance noted and then diminishes
    - range - distance at which the difference in values are no longer correlated
    - sill – average difference in value where there is no relationship between location and value
    - nugget – degree of uncertainty when measuring values for geographic locations that are very close to one another
      - Effect of sampling, measurement error, etc.
      - Unlikely that two samples near each other will have the exact same value

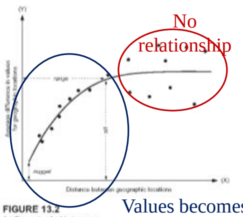
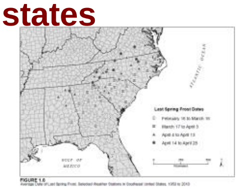


FIGURE 13.2
An Example of a Variogram

Variogram Example: Last Spring Frost IN SE United states

- Two nearby stations, LSF dates should be similar
  - 0 to 400 miles: distances between stations are large, dates are different
  - Beyond 400 miles, no longer spatially autocorrelated…



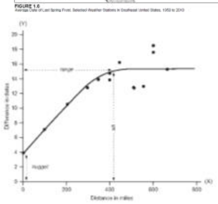


FIGURE 13.3

Spatial Autocorrelation: Importance in Geographic Research

- GIS – push of a button
  - Calculates relationship for any distances…
    - Is the test appropriate for any distance?
- Presence of spatial autocorrelation
  - Inferential statistics assume independent observations
    - Example: last spring frost dates are spatially correlated!
    - Impact: sample locations close together, just like taking the same sample
      - Sample size impacts size of standard error
        - » Smaller standard error than warranted
      - Standard deviation calculation impacted
        - » Even smaller standard error
- Global or local measurement
  - global – examine a distribution of subset (ex. ethnic group) across entire area (ex. city)
    - One group more clustered, dispersed or random than another
  - local – compares each geographic object (ex. all group members) with its surrounding neighbors
    - Is area (ex. neighborhood) more clustered, dispersed or random than another?

Spatial Autocorrelation: Neighbor Definitions

- Measure of interaction between geographic features
  - Defining neighbor…
    - adjacency – share common border
      - Binary: yes or no
        - » Ex. New York and Pennsylvania, New York and California
    - distance threshold – cut-off distance
      - Salisbury, MD – neighbor definition 60 miles…Easton, Wilmington, DE?
    - inverse-distance – strength of "neighborliness" between two objects as a function of distance separating them (1/distance)
        - » New York City and Boston: 1/189 miles or .005,
        - » NYC and LA: 1/2588 miles or .0004
          - » Interaction measure ("neighborliness") is 12 times stronger between NYC and Boston versus NYC and LA
  - In equations/modeling, takes the form of weights
    - w_{ij} : weight between geographic object *i* and *j*
      - Binary: 0 or 1
      - Inverse-distance: continuous value …

Spatial Autocorrelation

- First law of geography: "everything is related to everything else, but near things are more related than distant things" – Waldo Tobler
- Many geographers would say "I don't understand spatial autocorrelation" Actually, they don't understand the mechanics, they do understand the concept.

Spatial Autocorrelation

- Spatial Autocorrelation – correlation of a variable with itself through space.
  - If there is any systematic pattern in the spatial distribution of a variable, it is said to be spatially autocorrelated
  - If nearby or neighboring areas are more alike, this is *positive spatial autocorrelation*
  - *Negative autocorrelation* describes patterns in which neighboring areas are unlike
  - Random patterns exhibit *no spatial autocorrelation*

Why spatial autocorrelation is important

- Most statistics are based on the assumption that the values of observations in each sample are independent of one another
- Positive spatial autocorrelation may violate this, if the samples were taken from nearby areas
- Goals of spatial autocorrelation
  - Measure the strength of spatial autocorrelation in a map
  - test the assumption of independence or randomness

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Spatial Autocorrelation

- Spatial Autocorrelation is, conceptually as well as empirically, the two-dimensional equivalent of redundancy
- It measures the extent to which the occurrence of an event in an areal unit constrains, or makes more probable, the occurrence of an event in a neighboring areal unit.

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Spatial Autocorrelation

- Non-spatial independence suggests many statistical tools and inferences are inappropriate.
  - Correlation coefficients or ordinary least squares regressions (OLS) to predict a consequence assumes that the observations have been selected randomly.
  - If the observations, however, are spatially clustered in some way, the estimates obtained from the correlation coefficient or OLS estimator will be biased and overly precise.
  - They are biased because the areas with higher concentration of events will have a greater impact on the model estimate and they will overestimate precision because, since events tend to be concentrated, there are actually fewer number of independent observations than are being assumed.

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Indices of Spatial Autocorrelation

- Moran's I
- Geary's C
- Ripley's K

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Moran's *I* Index (Global)

- Popular technique for quantifying level of spatial autocorrelation in a set of geographic areas
- Moran's *I* Index takes into account geographic locations (points or areas) as well as attribute values (ordinal or interval/ratio) to determine if areas are clustered, randomly located or dispersed
  - Positive : clustered – nearby locations have similar attribute values
  - Negative: dispersed – nearby locations have dissimilar attribute values
  - Near zero: attribute values are randomly dispersed throughout study area

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Moran's *I* Index (Global)

The general form of Moran's Index for areas is shown in equation 15.4, and the mathematical equivalent, as presented in Ebdon (1988), appears in equation 15.5:

$$I = \frac{\begin{pmatrix}\text{number}\\ \text{of areas}\end{pmatrix}\begin{pmatrix}\text{sum of cross-products}\\ \text{for all contiguous pairs } (i,j)\end{pmatrix}}{\begin{pmatrix}\text{number}\\ \text{of joins}\end{pmatrix}\begin{pmatrix}\text{variance of the}\\ \text{area attribute values}\end{pmatrix}} \quad (15.4)$$

$$I = \frac{n\sum(x_i - \bar{x})(x_j - \bar{x})}{J\sum(x - \bar{x})^2} \quad (15.5)$$

where:

n = the number of areas
x = an area attribute value
$\bar{x}$ = the mean of all area attribute values
x_i and x_j = the values of contiguous pairs

$\sum(x_i - \bar{x})(x_j - \bar{x})$ = the sum of all contiguous pairs
J = the number of joins
$\sum(x - \bar{x})^2$ = the variance of the attribute values.

Weighted cross-products: deviation values for contiguous pairs multiplied together and summed

- Positive: neighboring areas with similar attribute values either large or small (clustered)
  - Larger deviation from mean, greater magnitude
- Negative: neighboring areas with dissimilar attribute values contiguous (dispersed)
  - Larger deviation from mean, greater magnitude
- Near zero: random…
- *I* ranges from -1.00 to 1.00

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Moran's *I* Index (Global): Significance test

The null hypothesis is that the area values are arranged in a completely random spatial pattern, and that spatial autocorrelation is not present within the study area. The Z score is computed as:

$$Z = \frac{I - E_I}{\sqrt{Var_I}} \qquad (15.6)$$

where: $E_I = \frac{-1}{(n-1)}$

Similar to the free and non-free sampling techniques mentioned with join count analysis and binary data, the variance for Moran's *I* may be calculated under the assumption of normality or randomization. *Normality* is similar to free sampling in that the observed values of a variable are taken randomly from a normally distributed population, whereas *randomization* is similar to non-free sampling in that the values are taken directly from the data set. The variance under the *normality* assumption is computed as:

$$Var_I = \frac{n^2J + 3J^2 - n\sum L^2}{J^2(n^2-1)}$$

where: n = the number of objects
J = the number of joins, and
$\sum L$ = the sum of the number of joins for each individual area

H_0: No spatial autocorrelation in the data (Values of areas are completely random)
H_A: Spatial autocorrelation in the data (Values of areas are not completely random)

- If p-value is not significant, then you should not reject the null hypothesis
  - The observed pattern is not different from complete spatial randomness
- p-value significant and *Z-score* ...itive ...stering
- ...ant and *Z-score* ...tive

Variance under the *randomization* assumption is computed as:

$$Var_I = \frac{n[J(n^2+3-3n)+3J^2-n\sum L^2]-k[J(n^2-n)+6J^2-2n\sum L^2]}{J^2(n-1)(n-2)(n-3)}$$

where: k represents the kurtosis for the variable x

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TABLE 15.6

Worktable for Moran's *I*: Five-area Example Pattern

H_0: no spatial autocorrelation in the data

H_A: spatial autocorrelation exists

Joins			Area values		
Area	L	L^2	x	$(x-\bar{x})$	$(x-\bar{x})^2$
A	2	4	70	25	625
B	3	9	80	35	1,225
C	3	9	20	−25	625
D	2	4	10	−35	1,225
Joins(J) = 5		$\sum L^2$ = 26	$\bar{x} = \frac{180}{4} = 45$		$\sum(x-\bar{x})^2$ = 3,700

Join number	x_i	$(x_i-\bar{x})$	x_j	$(x_j-\bar{x})$	$(x_i-\bar{x})(x_j-\bar{x})$
1	70	25	80	35	875
2	70	25	20	−25	−625
3	20	−25	80	35	−875
4	80	35	10	−35	−1,225
5	10	−35	20	−25	875
SUM					−975

$$E_I = \frac{-1}{(n-1)} = -\frac{1}{4-1} = -0.33$$

$$I = \frac{n\sum(x_i-\bar{x})(x_j-\bar{x})}{J\sum(x-\bar{x})^2} = \frac{4(-975)}{5(3{,}700)} = -0.21$$

$$Var_I = \frac{n^2J + 3J^2 - n\sum L^2}{J^2(n^2-1)} = \frac{(4^2\times 5)+(3\times 5^2)-(4\times 26)}{5^2(4^2-1)} = \frac{51}{375} = 0.136$$

$$Z = \frac{I-E_I}{\sqrt{Var_I}} = \frac{-0.21-(-0.33)}{\sqrt{0.136}} = 0.326$$

p = 0.74

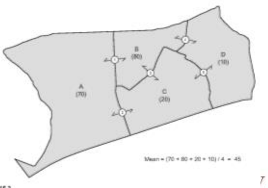

FIGURE 15.3
Areas and Attribute Values for Moran's *I*

Result?

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Example: Cleveland Census Block Groups

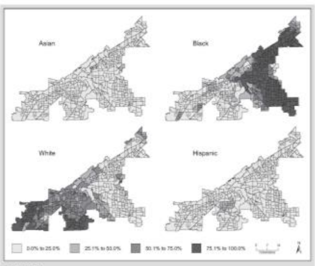

FIGURE 15.4
Distribution of Various Racial and Ethnic Groups by Census Block Group in Cleveland, Ohio

TABLE 15.7

Moran's *I* Values and Test Statistics for Selected Racial and Ethnic Groups in Cleveland, Ohio Census Block Groups

Group	Moran's Index	Z	p-value
Asian	0.35	18.39	0.000
Black	0.58	64.6	0.000
White	0.64	65.4	0.000
Hispanic	0.81	65.7	0.000

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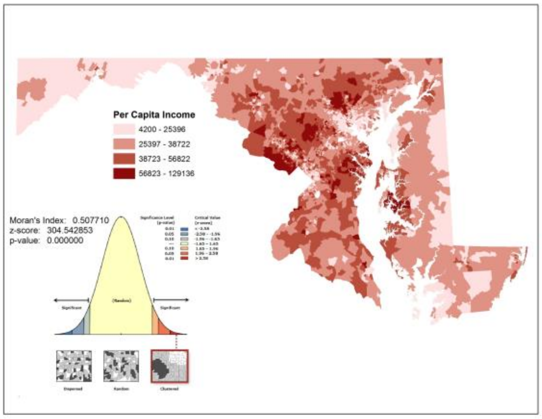
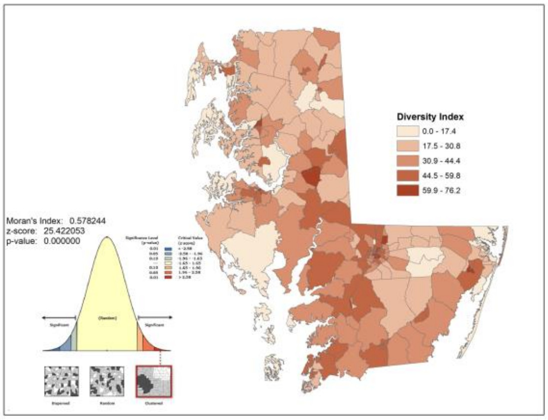
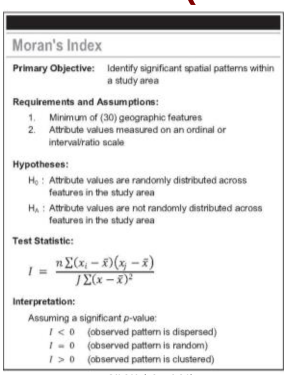




Moran's *I* Index (Global)



Moran's Index

Primary Objective: Identify significant spatial patterns within a study area

Requirements and Assumptions:

1. Minimum of (30) geographic features
2. Attribute values measured on an ordinal or interval/ratio scale

Hypotheses:

H_0 : Attribute values are randomly distributed across features in the study area

H_A : Attribute values are not randomly distributed across features in the study area

Test Statistic:

$$I = \frac{n \sum (x_i - \bar{x})(x_j - \bar{x})}{J \sum (x - \bar{x})^2}$$

Interpretation:

Assuming a significant *p*-value:

$I < 0$ (observed pattern is dispersed)

$I = 0$ (observed pattern is random)

$I > 0$ (observed pattern is clustered)

Moran's I Index (local)

- Global spatial autocorrelation (Moran's *I*) may indicate a lack of spatial autocorrelation
  - Local pockets may exist– hotspots
  - LISA – Local Indicators of Spatial Association
    - Quantify similarity of each geographic observation with an identified group of geographic neighbors
      - Identifies local clusters – geographic locations where adjacent or nearby areas have similar values
      - Spatial outliers – geographic locations that are different from adjacent or nearby areas
    - Each geographic area receives individual measure

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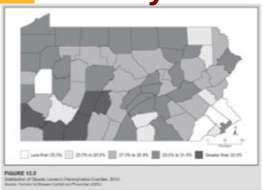
Moran's I Index (local): Example: Obesity in PA



$$I = \frac{(x_i - \bar{X})}{S_i^2} \sum_{j=1, j \neq i} w_{ij}(x_j - \bar{X}) \quad (15.7)$$

$$S_i^2 = \sum_{j=1, j \neq i} \frac{(x_j - \bar{X})^2}{n-1} - \bar{X}^2$$

where: x_i = the value for a particular geographic entity
x_j = the value for the neighboring geographic entity
$\bar{X}$ = the average of all attributes
w_{ij} = the spatial weights.

The overall computation of the local Moran index is beyond the scope of this book, but a short example will help illustrate its usefulness.

- Global Moran's *I* = .69, *p-value* = .25
- Local Moran's *I* for each county…

Positive values: similar levels in adjacent counties (clustering)

- Philly…
- Johnstown/Altoona

Negative values: dissimilar values – outlier

- Fayette County

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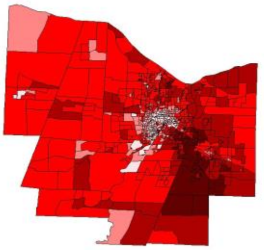
Example of Moran's I – Per Capita Income in Monroe County

Using Polygons:
Morans I: .66
P: < .001

Using Points:
I: .12
Z: 65



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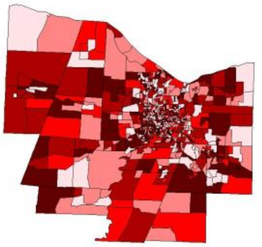
Example of Moran's I – Random Variable

Using Polygons:

Moran's I: .012
p: .515

Using Points:

Moran's I: .0091
Z: 1.36



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