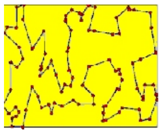


"The problem of designing a good or efficient route set (or route network) for a transit system is a difficult optimization problem which does not lend itself readily to mathematical programming formulations and solutions using traditional techniques".Given the importance of devising faster methods to obtain optimal/near-optimal solutions to the routing and scheduling problems a lot of work has been going on to develop such techniques using new tools like Tabu search, Simulated annealing, Ant Systems, and Genetic Algorithms.

Optimal Routing and Scheduling in Transportation: Using Genetic Algorithm to Solve Difficult Optimization Problems

By Partha Chakroborty

1. Introduction

Whenever an organization, in the business of providing mobility, is entrusted with moving goods and people a natural question that arises is how efficiently can that organization provide the services. This basic requirement of efficient mobility of goods and passengers gives rise to, among many other things, the subject areas of optimal routing and scheduling. In the following sections the problems of optimal routing and optimal scheduling are explained. Finally, how these optimization problems, which are often difficult to solve using traditional optimization tools, have been solved using genetic algorithms are explained.

2. The Optimal Routing Problem

The problem here is to find a path which achieves some pre-defined purpose and is desirable (i.e., it is optimal or good in some way). There are two major classes of routing problems, namely the vehicle routing problem, and the transit (or bus) routing problem. Figure 1, for example, shows a part of the Vishakapatnam road network with the bus route system superimposed (in purple); the job of optimal transit routing would be to determine such a bus system which is optimal from various different standpoints (described later).

The sheer number of different possible routes and various different constraints representing

Figure 1: Vishakapatnam road network with the bus routes.

several resource limitations make the development of such a bus system difficult.

2.1. The vehicle routing problem

The vehicle routing problem refers to all problems where optimal closed loop paths which touch different points of interest are to be determined. There may be one or more vehicles. Generally the points of interest are referred to as nodes; further, the start and end nodes of a route are the same and often referred to as the depot.

Broadly, there are six sub-classes of the vehicle routing problem; these vary from one another depending on the node and vehicle properties. Historically, many of these problems have specific names which have been used here. These problems are described briefly in the following text.

The traveling salesman problem (TSP)

In this case a single vehicle has to visit a set of nodes exactly once before returning to its starting position. Such problems implicitly assume that the sum total of demand for services at the nodes is less than the capacity of the vehicle, or alternatively the capacity of the vehicle is not material to the problem. In this case optimality of a route is measured in terms of minimum route length. Practical examples of the TSP include planning the route for a courier who typically has to visit certain homes / houses in an area; other examples include that of developing a repairman's route, or that of a doctor making house calls. More importantly the TSP often forms a sub-problem of other vehicle routing problems. As an example of the TSP, Figure 2 shows two possible routes of a courier serving five offices. Both the routes are viable or feasible; yet it is clear that the route shown in (b) part of the figure is desirable (when compared to (a)) as the length of the route connecting all the five offices is less than the length of the route shown in (a). The purpose of TSP algorithms is to find that route which offers the least length (or at least find a route which from a practical stand point is as good as the minimum length route).

The TSP is a difficult optimization problem as the number of feasible routes (from which the best is to be found) increases at a very fast rate with the increase in the number of nodes. Nonetheless, some exact algorithms exist (see Padberg and Rinaldi [35]) which solve the TSP using polyhedral cutting plane procedures. However, the computation effort is extremely large and the process complexity (i.e., the complexity of the algorithms and their implementation) is prohibitively large. Similar observations are made by, among others, Chatterjee et al. [16] and Hasegawa et al. [24]. It is not surprising, therefore, that even with the existence of exact algorithms, ever more efficient heuristics continue to be developed and reported (among the recent ones are Hasegawa [24], Ugajin [45], del Castillo [18], etc.). Most of the recent heuristic algorithms are based on what Fisher [21] calls artificial intelligence techniques, like Tabu search, Simulated Annealing, Neural Networks, and Genetic Algorithms.

The single vehicle pick-up and delivery problem (SVPDP)

This problem is similar to a TSP except that, each node is either a pick-up node or a delivery node; further, there is a one-to-one, one-to-many, many-to-one, or many-to-many relation between

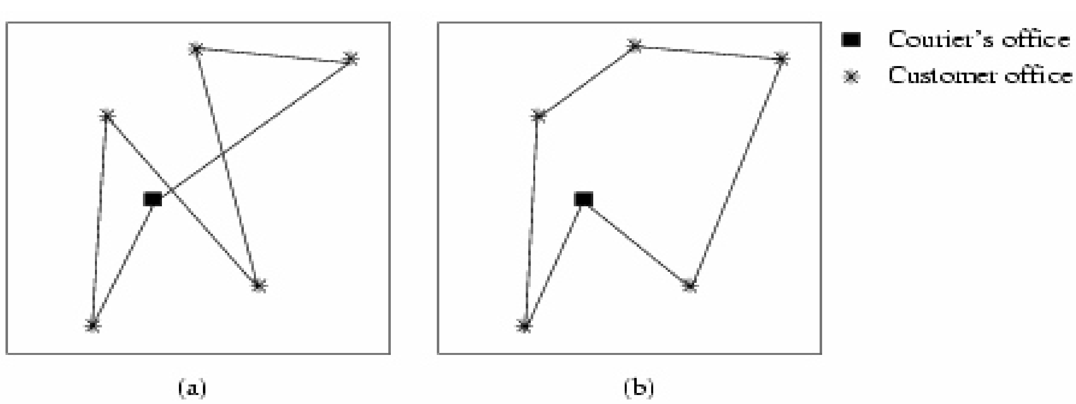


Figure 2: The TSP; (a) a feasible path through 5 nodes,

(b) a better and feasible path through the same 5 nodes.

the pick-up node set and the delivery node set. Obviously, a sequence of nodes where a delivery node appears before its corresponding pick-up node is not a valid route. As in the TSP, each node can have different service requirements. Such problems arise in situations where intra-city courier service personnel must pick-up and deliver mail among various offices in a city, or in situations where a garbage truck must leave from depot collect garbage, deposit it at a dump and then return to the depot, etc. Route length is an important optimality criterion in such problems; however, the riding time of goods (or people) between the pick-up point and the delivery point can in some cases be the optimality consideration. This problem is sometimes referred to as a traveling salesman problem with precedence constraints because there are constraints on how nodes can be ordered (a pick-up node must be before the corresponding delivery node).

Single vehicle pick-up and delivery problem with time windows (SVPDPTW)

This problem is same as the SVPDP except that there is a time-window associated with each node. The vehicle serving a particular node must visit that node within the stipulated time-window. In this problem, therefore, a sequence of nodes cannot be considered as a valid route if (i) a delivery node is visited before its corresponding pick-up node, and (ii) a node is not visited within the specified time window. A good example of this problem is the dial-a-ride para-transit system where individuals ask the service provider to pick them up from a certain point within a certain time and drop them off at another point within a certain time window. The total route length is an important optimality criterion in these problems. Riding time is not as important since satisfaction of time windows imply, to a certain extent, the satisfaction of users from the riding time perspective.

Much less work (as compared to TSP) has been reported in the last two types of vehicle routing problem, namely the pick-up and delivery problem, and the pick-up and delivery with time window problem. Savelsbergh and Sol [41] and Renaud et al. [38] make similar observations. As earlier, even though there are some exact methods (like Kalantari et al.'s [26] branch-bound procedure for pick-up and delivery problems and Psaraftis' [36] dynamic programming procedure for SVPDPTW) their applicability is limited owing to their complexity. Hence, heuristic solution techniques continue to be developed. Among the recent ones are Moon et al. [32], Renaud et al. [39,38], and Gendreau et al. [22] for the pick-up and delivery problem and Calvo [6], and Nanry and Barnes [33] for the pick-up and delivery problem with time windows.

Multiple vehicle routing problems

It is conceivable that in each of the above cases, the total of the services (or goods) demanded by all the nodes is greater than the capacity of one vehicle. In this case, more than one vehicle needs to be used. Although the criterion for optimization can remain the same as in the corresponding single vehicle case the multiple vehicle problems is in essence different from the single vehicle case. The difference arises because, as opposed to the single vehicle case, here, one is not sure which nodes need to be served by a given vehicle. That is, a priori, one does not know which nodes a route should touch; all that is known is that all the routes put together should serve all the nodes in the problem. Typically, in these problems, it is assumed that complete service at a node must be provided by one vehicle; part service of a node is not allowed.

Quite a lot of work has been done on multiple vehicle routing problems; these are not discussed here. However, the interested reader may refer to Fisher [21], Desrosiers et al. [19], Laporte at al. [28], Toth and Vigo [43], or Tan et al. [42] for descriptions of the various concepts and models used in solving the different kinds of multiple vehicle routing problems.

2.2. The transit routing problem

The transit routing problem is quite distinct from the vehicle routing problems. In transit routing, a route is to be determined on which transit units (say buses) will run as per some pre-defined (and possibly announced) schedule. Figure 3 shows in part (a) a typical urban area with a road network and the underlying land-use pattern; part (b) of the figure shows a possible bus route set for the area. The purpose of transit routing is to determine a good set of routes.

Transit routing is different from the vehicle routing problems described earlier because (i) the vehicle being routed, in this case a bus, does not have to visit the actual points of demand (i.e., the points where the demand arises), rather the points of demand re-adjust themselves (by gathering at a bus stop) to avail of the services; (ii) the demand need not be satisfied using one route, one can transfer (from one route to another) in order to reach ones final destination; and (iii) it is not necessary that all demand for travel be met. Given these differences, the criteria which define a good route, or more correctly, a good set of routes are different from the vehicle routing problems. In this case, a good set of routes should have the following properties:

(i) The route set should satisfy most, if not all, of the existing transit demand (i.e., the requirements of the people to travel);

(ii) The route set should satisfy most of the demand without requiring passengers to transfer from one route to another;

(iii) The route set should offer low travel time (including the time spent by passengers in transferring) to its passengers.

The problem of designing a good or efficient route set (or route network) for a transit system is a difficult optimization problem which does not lend itself readily to mathematical programming formulations and solutions using traditional techniques. Newell [34] observes that designing an efficient route network "... is generally a non-convex (even concave) optimization problem for which no simple procedure exists short of direct comparisons of the various local minima." Similar observations are also made by Baaj and Mahmassani [1].

These reasons, perhaps, have limited the solution of this problem to either using heuristic algorithms or analytical techniques which optimize only parameters like route spacing, route length, etc. for simplistic, idealized networks. The analytical techniques (example, Holroyd [25], Byrne and Vuchic [4], and Byrne [5]), as also observed by Ceder and Wilson [7] and van Nes et al. [46], cannot be used for designing of actual routes on any given road network. As mentioned earlier, most of the other

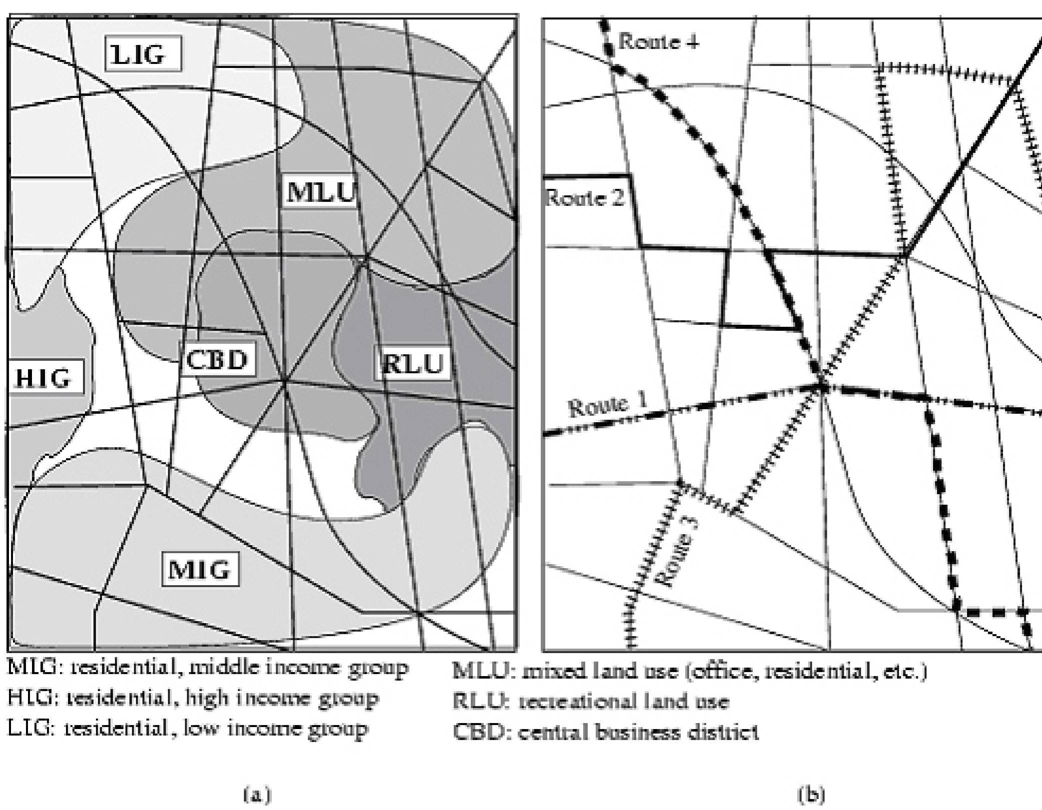


Figure 3: The transit routing problem; (a) an urban area with land use and roads, (b) a possible bus route network for the urban area.

studies in the area of route design like Lampkins and Saalmans [29], Rosello [40], Mandl [30,31], Dubois et al. [20], Ceder and Wilson [7], and Baaj and Mahmassani [1,2] basically propose heuristic algorithms at various levels of sophistication. Recently, Kidwai [27] has made an attempt to use an optimization tool for solving the problem.

In this section on routing problems various types of problems have been presented. These problems vary widely in terms of their purpose, the characteristics of the nodes, and the vehicles. However, three things remain common: (i) in all cases the geometry of a path is sought, (ii) the geometry should be optimal or near optimal from some perspective, and (iii) all of them are discrete, difficult (NP hard), combinatorial optimization problems. Problems which possess the latter characteristic are notoriously difficult to solve using traditional optimization techniques. Later, a series of algorithms developed at IIT Kanpur to solve various routing problems will be briefly described.

3. The Optimal Scheduling Problem

Another optimization problem related to the transit system design is the scheduling of transit units (say buses). The problem here is that given a set of routes, one needs to develop schedules for bus arrivals and departures at all the stops of the network. A good or efficient schedule is one which minimizes the waiting time of passengers while operating within a set of resource and service related constraints. The total waiting time of passengers have two components: (i) the total initial waiting time (IWT) of passengers, this is the sum of the waiting times of all the passengers at their point of origin, and (ii) the total transfer time (TT), this is the sum of the transfer times of all the transferring passengers. The resource and service related constraints are:

1. Limited fleet size: only a fixed number of buses are available for operating on the different routes.
2. Limited bus capacity: each bus has a finite capacity.
3. Stopping time bounds: buses cannot stop for a very little or a very long time at a stop.
4. Policy headway: on a given route a minimum frequency level needs to be maintained.
5. Maximum transfer time: no passenger should have to wait too long for a transfer.

Some of the features related to the transit scheduling problem which any methodology designed to solve the problem must be capable of handling are:

1. Arrival time of a bus at one stop is dependent on the arrival time of the bus at the previous stop.
2. Arrival times of buses at a stop are generally not exactly as per the schedule. Arrival times are generally randomly distributed around the scheduled arrival times. Since arrival times are not exactly as per schedule, departure times are also not exactly as per schedule.
3. If demand for a route is very high during a particular period, then the queue developed for that route at the stop may not be cleared entirely by the next bus of the route due to limited bus capacity. In such cases, the formation and the dissipation of the queue must be tracked so that realistic values for the waiting times and transfer times can be obtained.
4. The arrival patterns in passengers at stops may vary widely; stops which primarily have commuters may see a surge in passenger arrivals just before the arrival time of a bus (since schedule is known); whereas stops which have a large percentage of irregular passengers may see a reasonably uniform arrival rate.

Unlike in the transit routing problem, the scheduling problem can be formulated as a mathematical programming (MP) problem. One may refer to Chakroborty [13] or Chakroborty et al. [9] for variants to the mathematical formulation. The MP formulation arising in this case is a mixed integer (i.e., some decision variables are integer while others are real) non-linear programming problem (MINLP); non-linearity exists both in the objective function and constraints. It must be mentioned here, that MP formulations make two important simplifying assumptions of unlimited bus capacity, and strict schedule adherence (or deterministic arrival times).

Given the combinatorial nature of the problem, the number of variables (especially the integer ones) and the number of constraints increase at a fast rate with the increase in the number of routes and fleet size. Further, given the restricted ability of traditional optimization methods to handle MINLP problems, it is seen that even extremely small problems (for example, three routes and ten buses in each route) cannot be solved within a reasonable time frame using traditional methods. As in the case of transit routing, most of the earlier work on transit scheduling with transfer considerations (for example, see Bookbinder and Désilets [3] and Rapp and Gehner [37]) rely on heuristics and user intervention at various stages of the solution process.

Before leaving the sections on routing and scheduling problems few points are worth highlighting:

1. The size of the optimization problem in all the types of routing and scheduling problem increases much more quickly than the rate at which the size of the actual problem increases. For example, if there are three nodes, there is just one feasible TSP route; now try finding how many feasible routes are there if there are 15 nodes.

The impact of this property is that even if exact solution techniques exist many of them are rendered useless in practical situations because of the excessive time requirements.

2. All problems deal with discrete quantities; dealing with such quantities using traditional techniques is difficult.

3. From practical standpoints the optimal is not necessarily one which has to be obtained; solutions which are good (i.e., close to the optimal) are equally important provided these can be obtained quickly. Not surprisingly therefore, heuristic techniques (which typically guarantee near-optimal solutions) continue to play an important role as viable solution techniques for these problems.

4. GA and Routing and Scheduling Problems

Given the importance of devising faster methods to obtain optimal / near-optimal solutions to the routing and scheduling problems a lot of work has been going on to develop such techniques using new tools like Tabu Search, Simulated Annealing, Ant Systems, and Genetic Algorithms. Over the last decade or so, IIT Kanpur has contributed to this effort by developing various methods using Genetic Algorithms. These methods can be found in the following articles: Chakroborty and Samanta [15], Chakroborty and Mandal [14], Chakroborty [13], Chakroborty and Dwivedi [12], Chakroborty et al. [11], Deb and Chakroborty [17], Chakroborty et al. [9], Chakroborty et al. [10], and Chakroborty et al. [8].

In the rest of this section certain results obtained from the various algorithms developed at IIT Kanpur are presented. For a detailed understanding of the algorithms the reader may refer to the relevant publications.

4.1. Results from GA based optimizer for vehicle routing problems

In Chakroborty and Mandal [14] a single GA based algorithm called ROUTER was proposed for solving TSP, SVPDP, and SVPDPTW. It was shown that this algorithm was faster than similar algorithms proposed earlier (see Chatterjee et al. [16]). Further, this is the only algorithm which can handle all the three types of problems mentioned above; all other methods are problem specific and not as general as ROUTER. The greatest strengths of this algorithm are its simplicity and its speed; results show that this algorithm is about 30 times faster than an existing fast GA based algorithm (see Chatterjee et al. [16]) with only a marginal decrement in solution quality (ROUTER solutions are at the most 0.8% higher than Chatterjee et al.'s [16] solutions).

Figure 4 shows a benchmark 70 node TSP (Part (a)) reported in the literature [44]. Part (b) shows a near-optimal solution obtained from ROUTER for the same problem. The near-optimal solution shown in Part (b) has a route length which is 1.0074 times the known optimum route length. Similar routes for other problems can be found in Chakraborty and Mandal [14].

Figure 5 shows the result obtained for a multiple vehicle routing problem using a GA based procedure developed in Chakroborty and Samanta [15]. In this benchmark problem, known as Eil51 (see [44]), there are no precedence constraints or time windows; each node only demands certain units and a vehicle has a finite capacity much lesser than the total demand of all the nodes. The solution shown in

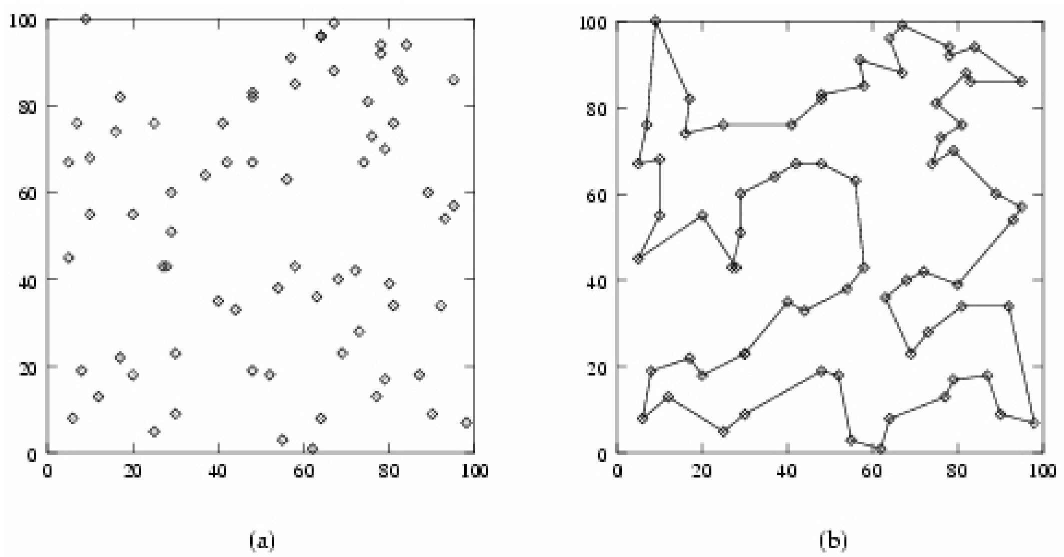


Figure 4: Result from St70: (a) the distribution of nodes in St70 (a benchmark problem) (b) the near-optimal route as obtained by ROUTER; route length 1 0074 optimal length.

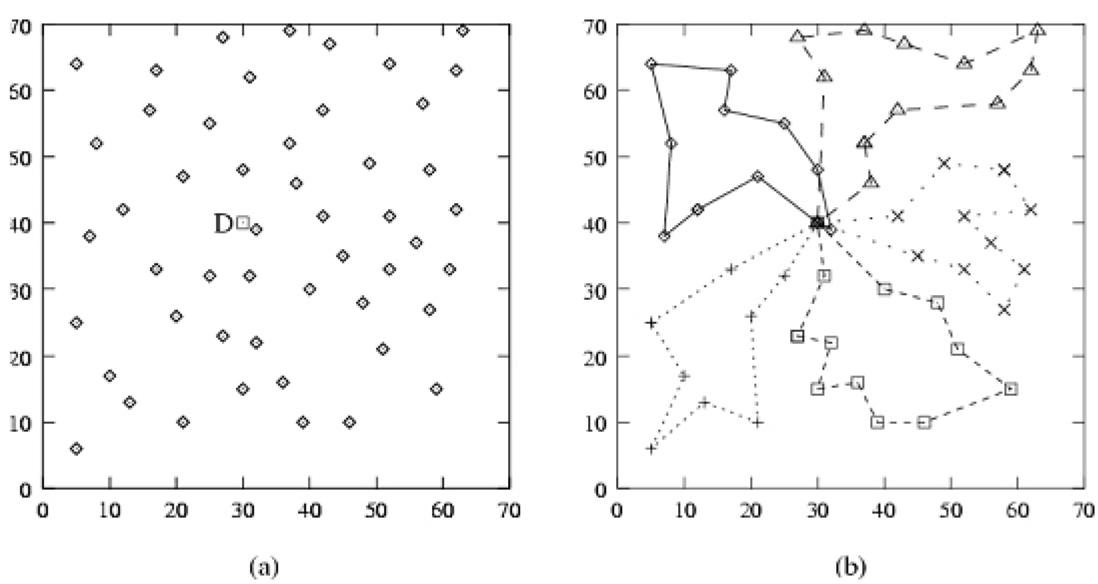


Figure 5: Multiple vehicle routing; (a) The distribution of nodes in the benchmark problem Eil51, (b) the near optimal route found using IIT Kanpur's GA based method ; total route length = 1.0055× the shortest route length reported for this problem.

Figure 5 uses five vehicles (note that there are five routes each denoted by a different line style) and has a total route length which is only 1.0055 times the best solution known for this problem.

4.2. Results from GA based optimizer for the transit routing problem

In Chakroborty and Dwivedi [12] a GA based algorithm for transit route design was developed. Figure 6 shows a set of four routes designed by the Chakroborty-Dwivedi algorithm on Mandl's network- a benchmark problem. The algorithm was used to determine various other route sets with different number of routes.

A comparison between the performances of the route set obtained here with those obtained, for the same number of routes, by Mandl [30], Baaj and Mahmassani [1] and Kidwai [27] is provided in Table 1. In the table, in a cell NR indicates that the result for that particular cell was not reported by the author. The comparison presented in the table uses the following measures of effectiveness:

- d_0^p , the percentage of demand satisfied directly by the route set,
- d_1^p , the percentage of demand satisfied with one transfer by the route set,
- d_2^p , the percentage of demand satisfied with two transfers by the route set,
- d_{un}^p , the percentage of demand unsatisfied by the route set,
- TS , the average travel time (including transfer penalty) per user in minutes, and
- ATT , the total man-hours saved per day by using the route set designed by the proposed algorithm instead of the route set reported in the literature.

It can be seen from the table (next page) that the route set obtained using the proposed algorithm offers a substantially lesser average travel time (ATT) than the route networks proposed by any of the other models. This results in substantial total time savings (TS). For example, for the four routes case, using the proposed route set instead of the one suggested by Mandl [30] produces a total saving (see last column of Table 1) of 259.5 man-hours per day (note that the total demand is 15570 trips per day). Similarly, use of the proposed route set instead of the one suggested by Kidwai [27] produces a total saving of 213 man-hours per day. Also note that in most cases the proposed route set satisfies a much larger percentage of demand directly (d_0^p in the table) a feature desirable in any route network design. These observations indicate the superiority of the proposed route network design algorithm.

4.3. Results from GA based optimizer for the scheduling problem

Substantial amount of work in this area has

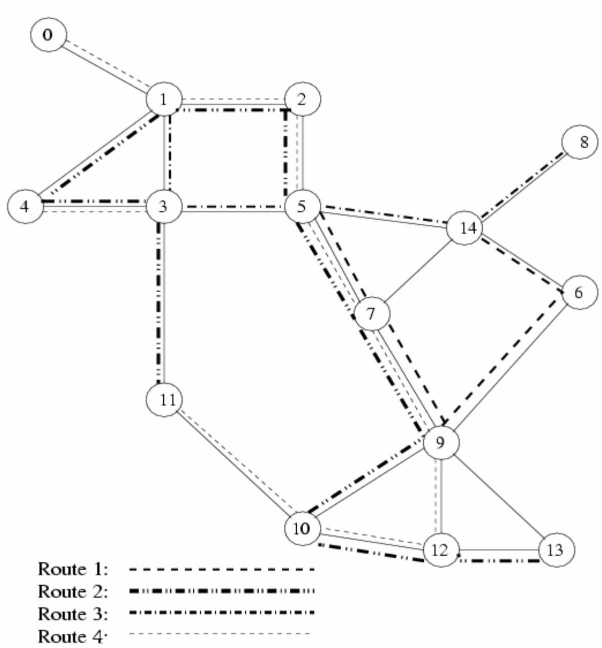


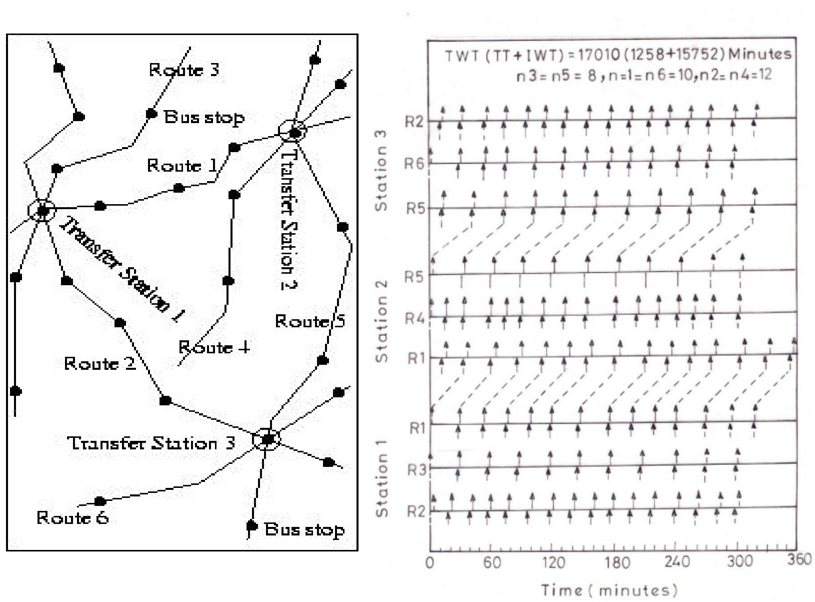
Figure 6: The "optimal" route set for Mandl's network with four routes in the set

Models	d_0^p (%)	d_1^p (%)	d_2^p (%)	d_{un}^p (%)	ATT (mpu[†])	TS (mhd[‡])
Four routes in route set (same as in Figure 6)						
Mandl [30]	69.94	29.93	0.13	0	12.9	259.5
Baaj & Mah. [1]	NR	NR	NR	NR	NR	NR
Kidwai [27]	72.95	26.91	0.13	0	12.72	213
Proposed Algorithm	86.86	12	1.14	0	11.9	—
Six routes in route set						
Mandl [30]	NR	NR	NR	NR	NR	NR
Baaj & Mah. [1]	78.61	21.39	0	0	11.86	405
Kidwai [27]	77.92	19.62	2.4	0	11.87	407
Proposed Algorithm	86.04	13.96	0	0	10.3	—
Seven routes in route set						
Mandl [30]	NR	NR	NR	NR	NR	NR
Baaj & Mah. [1]	80.99	19.01	0	0	12.5	610
Kidwai [27]	93.91	6.09	0	0	10.7	143
Proposed Algorithm	89.15	10.85	0	0	10.15	—
Eight routes in route set						
Mandl [30]	NR	NR	NR	NR	NR	NR
Baaj & Mah. [1]	79.96	20.04	0	0	11.86	363
Kidwai [27]	84.73	15.27	0	0	11.22	197
Proposed Algorithm	90.38	9.58	0	0	10.46	—
†mpu: minutes per user				‡mhd: man-hours per day		

Table 1: Comparison of route sets with different number of routes

solutions to scheduling problems with transfer considerations. Scheduling problems with stochastic arrival times and finite bus capacities have also been looked at.

In this section only one schedule developed for the network shown in Figure 7 (a) is given in part (b) of the same figure. The schedule states the arrival and departure times of buses at the transfer stops only. The schedule at the other stops can be deduced from the above schedule easily. Other schedules which help in establishing the quality of the GA based solutions can be seen in the articles cited in this section.



(a) (b)

Figure 7: "Optimal" Schedule of buses on six routes (shown in (b)) plying on the network shown in (a); the number of buses in Route i is given as ni in the figure, TWT refers to the sum of all the initial waiting times (IWT) and transfer times (TT) of all the passengers.

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