Measurement of Coriolis Acceleration with a Smartphone

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Undergraduate physics laboratories seldom have experiments that measure the Coriolis acceleration. This has traditionally been the case owing to the inherent complexities of making such measurements. Articles on the experimental determination of the Coriolis acceleration are few and far between in the physics literature.[1,2] However, because modern smartphones come with a raft of built-in sensors, we have a unique opportunity to experimentally determine the Coriolis acceleration conveniently in a pedagogically enlightening environment at modest cost by using student-owned smartphones. Here we employ the gyroscope and accelerometer in a smartphone to verify the dependence of Coriolis acceleration on the angular velocity of a rotating-track and the speed of the sliding smartphone.

What is the Coriolis force?

The Coriolis force[3] is a fictitious force observed by an individual in a rotating frame of reference (also known as a non-inertial frame of reference). An excellent elementary treatment of Coriolis force can be found in *The Feynman Lectures on Physics*.[4] Here we will provide a synopsis of Feynman's explanation of Coriolis force. An individual in a rotating frame of reference observes "forces" acting on objects that are not real forces. These "forces" are called fictitious forces. The simplest example of a fictitious force is centrifugal force. If an individual seated on a rotating carousel places an object on the floor, the object will appear to slide radially outward (assuming negligible friction). This will be attributed by the individual to "centrifugal force." The formula for "centrifugal force" as calculated by the individual on the rotating carousel is given by $m\omega^2 r$, where m is the mass of the object, ω is the angular velocity of the carousel, and r is the radial distance of the object from the axis of rotation. In reality, there is no real force acting on the object. A person in the nonrotating inertial frame sees the same object as simply moving tangentially from its point of release with constant speed and direction as one would expect from Newton's first law. In the inertial frame, there is no real force needed to explain this motion; in the rotating frame, often the introduction of fictitious forces allows ready solutions to otherwise unwieldy problems.

In addition to the centrifugal force, there is another fictitious force that is observed by an individual in a rotating frame of reference. An object that is moving radially is seen to veer away from the radial path by this observer. The fictitious force causing this deviation from the expected path goes by the name Coriolis force. Again this is not a real force and appears so only because the individual happens to be on a rotating platform.

Here we will give an elementary derivation of the formula for the Coriolis force. The angular momentum of our object on the floor of the carousel (rotating platform) is given as $L = m\omega r^2$.

As the object slides out radially, the individual on the carousel realizes that r is increasing and so is the angular momentum:

$$dL/dt = 2m\omega r\, dr/dt, \text{ or}$$

$$dL/dt = 2m\omega rv.$$

Angular momentum changes if a torque is acting on the object. So our individual on the carousel concludes that the torque on the object is $\tau = 2m\omega rv$. Now $\tau = Fr$, so the Coriolis force is $F = 2m\omega v$ and the Coriolis acceleration is $a = 2\omega v$.

The general form of this equation for situations where ω and v are not perpendicular involves a cross-product $\mathbf{a} = 2\boldsymbol{\omega} \times \mathbf{v}$.

Experimental procedure

In this section we show how a smartphone can be used to measure the Coriolis acceleration. A nearly semicircular symmetric track was molded from foam board and a low-friction laminate was glued onto it (Fig. 1). The whole contraption is inexpensive, very lightweight, and has a diameter of approximately 1.5 m. An electric drill was mounted at the center-bottom and used to spin this track at various angular velocities. It is not necessary to have electronic speed control of the rotating track. The record button on the app[5] was pressed and the smartphone was subsequently released from four different heights. The time delay does not have a deleterious effect on the integrity of the measurements as the following discussion will convince the reader. The triple-axis gyroscope and accelerometer outputs from

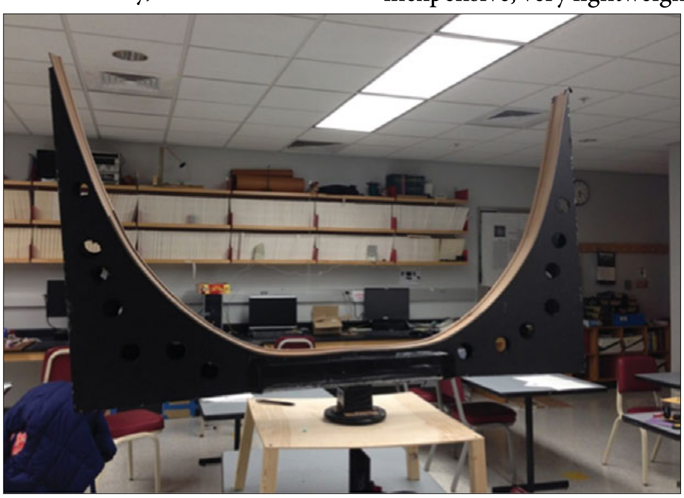

Fig. 1. Smartphone sliding down the Coriolis acceleration track.

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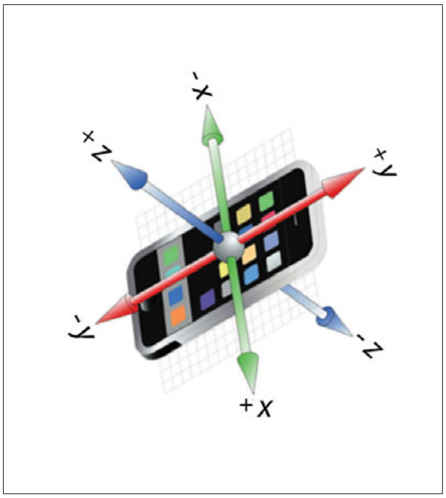

Fig. 2. *x*-, *y*-, and *z*-axes on the smartphone.

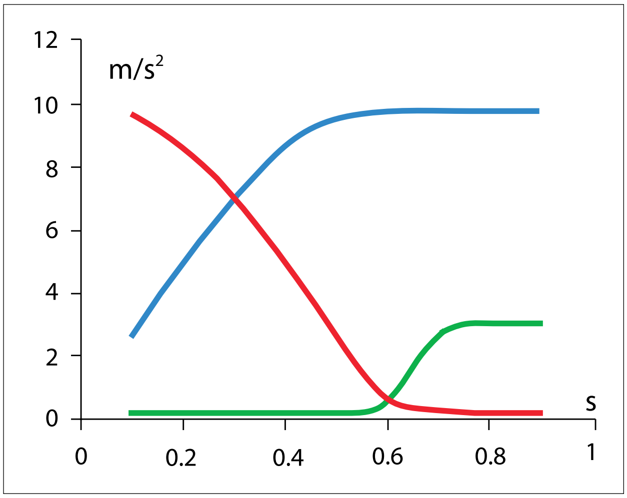

Fig. 3. Typical acceleration-vs.-time plot for our sliding smartphone. As in Fig. 2, green, red, and blue correspond to the *x*-, *y*- and *z*-axes respectively.

the smartphone were recorded every tenth of a second by the app for some 20 seconds. The gyroscope output gives ω_x, ω_y, and ω_z, and the accelerometer output gives a_x, a_y, and a_z. The smartphone slides along its long side, which has been designated by the manufacturers as the y-axis. The shorter side is the x-axis and the axis perpendicular to the screen of the smartphone is the z-axis (Fig. 2). When the smartphone is sliding along the flat section of the track, the drill spins the track about the z-axis and the Coriolis acceleration is along the x-axis. Before the actual data are collected, a quick calibration is undertaken. The smartphone is taped to the center of the track and the track is not rotated. The app button on the smartphone is pressed. The output data from the smartphone should show that ω_x, ω_y, and ω_z are very close to zero. Also, a_x and a_y will read close to zero and a_z will read close to 1g (9.8 m/s^2). Then the drill is turned on and the same calibration procedure is repeated. This time the output of the smartphone will indicate that ω_z has some finite value, say, 1.5 rad/s. This is verified by means of a stopwatch to ensure that the output of the gyroscope is correct. The next step is to measure the speed of the smartphone as it traverses the flat section of the stationary track (not spinning). The smartphone is released from four different heights and the time to traverse the flat section of the track is measured. The speed is given by $v = d/t$, where d is the length of the flat section and t is the time to travel the distance d. This is repeated three times for each of the four heights. The speed v is also calculated by an alternate procedure that uses the time stamp of the smartphone itself. As indicated previously, entry of the smartphone into the flat section of the track is established when a_x and a_y read close to zero and a_z reads close to 9.8 m/s^2. A composite sketch of acceleration vs. time is depicted in Fig. 3. The smartphone slides down the stationary track from 0 to 0.6 s in Fig. 3 and then slides along the flat section of the spinning track after that. Exit from the flat section is established when the condition that a_y reads close to zero and a_z reads close to 9.8 m/s^2 is not true any longer. The elapsed time between these events is read from the time stamp generated by the app. The average value of the speed of the smartphone in the flat section is thus established fairly accurately.

At this point we are ready to collect the actual data for the Coriolis acceleration. The track is spun at several different angular velocities about the z-axis and the smartphone is released from four different heights. The Coriolis acceleration is measured only when the smartphone is moving in the flat section of the track. In order to confirm the Coriolis acceleration acting on the smartphone as it traverses the flat section of the rotating track, we need to know the speed of the smartphone in the flat section (v), the angular velocity of the rotating track (ω_z), and the x-component of the acceleration (a_x). The measurement of the speed of the smartphone (v) was described earlier in this section. The quantities of interest to us from the output of the smartphone are a_x and ω_z. The sliding smartphone in the flat section has a speed v along the y-axis on a track spinning about the z-axis and experiences a Coriolis acceleration a_x.

Experimental data

We have tabulated the data for the Coriolis acceleration in Table I. The speed of the smartphone as it traverses the flat section of the spinning track is along the y-axis and is denoted as v. The angular velocity of the track spinning about the z-axis is denoted as ω_z. Note that it is not at all necessary that we maintain the same value of ω_z because the accelerometer and gyroscope are collecting the data every tenth of a second. We know a_x and ω_z every tenth of a second. Indeed, the gyroscope output gives ω_x, ω_y, and ω_z, and the accelerometer output gives a_x, a_y, and a_z every tenth of a second for 20 seconds, and we can create a veritable history of the entire motion should we feel so inclined.

Experiment meets theory

We will analyze the tabulated data and perform a sample calculation. The speed v of the smartphone in the y-direction (along the long side of the smartphone) is perpendicular to the angular velocity of the spinning track ω_z in the z-direction. Thus, the Coriolis acceleration experienced by the smartphone, a_x, is in the x-direction, along the short side of the smartphone. The formula for Coriolis accelerations is given as a cross-product, which only has an x-component here:

$$\mathbf{a} = 2\boldsymbol{\omega} \times \mathbf{v} = 2|\omega_z|\,|v|\,\hat{\mathbf{i}}.$$

From Table I, for v = 1.3 m/s and ω_z = 2.5 rad/s,

Table I. Gyroscope and accelerometer data for smartphone sliding down a spinning track.

v (m/s) average	ω_z (rad/s)	a_x (m/s^2) (measured)	a_x (m/s^2) (calculated)	(a_x measured / a_x calculated)
1.3	1.9	4.9	4.9	1.0
1.3	2.1	4.5	5.5	0.80
1.3	2.2	4.9	5.7	0.86
1.3	2.5	6.9	6.5	1.1
1.8	0.9	3.2	3.2	1.0
1.8	1.0	3.9	3.6	1.1
2.3	0.77	3.2	3.5	0.91
2.3	0.83	3.4	3.8	0.89
2.3	1.2	5.6	5.5	1.0
2.3	1.6	7.8	7.4	1.1
2.7	1.4	6.7	7.6	0.88
2.7	2.5	14.3	13.5	1.06

$a_x = 6.5$ m/s^2. This compares favorably with the accelerometer's output of 6.9 m/s^2. The last row in Table I gives for $v = 2.7$ m/s and $\omega_z = 2.5$ rad/s, $a_x = 13.5$ m/s^2. The smartphone's accelerometer displays 14.3 m/s^2. The ratio a_x measured/a_x calculated is shown in Table I. The average deviation is only 2.5%.

Measurement uncertainties

Perhaps the greatest source of uncertainty is in the estimation of the speed (v) of the smartphone as it traverses the flat section. The laminate on which the smartphone slides has very low friction. However, it is clear that any sliding object will slow down as it slides. We have measured the average speed of the smartphone by two different methods. One is the traditional method of dividing the distance of the flat section by the time taken. The other method is the time stamp generated by the smartphone. As indicated previously, entry of the smartphone into the flat section of the track is established when a_x and a_y read close to zero and a_z reads close to 1g (9.8 m/s^2). Exit from the flat section is established when this condition is not true any longer. The elapsed time between these events is read from the time stamp generated by the app. The average value of the speed of the smartphone in the flat section is thus established fairly accurately.

Conclusion

Smartphones are now routinely used in collecting sensor data for physics experiments.[6-10] In this paper we simultaneously used the outputs of a smartphone's triple-axis gyroscope and triple-axis accelerometer to measure the Coriolis acceleration experienced by a smartphone sliding in the flat section of a spinning track. The results compare favorably with the theoretical values. The smartphone is a robust and versatile device that can accurately, conveniently, and reproducibly measure physical quantities such as magnetic fields, acceleration, and angular velocity. This creates an opportunity for a new paradigm in physics pedagogy. Student-owned smartphones can conveniently be implemented in the physics laboratory while concomitantly reducing the laboratory budget. We have found that students take enormous pride in the data generated by their smartphones and are excited and motivated to learn from them. We even let them take their newfound physics toy home with them!

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