Bullet-Block Science Video Puzzle

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A science video blog,[1] which has gone viral, shows a wooden block shot by a vertically aimed rifle. The video[2] shows that the block hit dead center goes exactly as high as the one shot off-center. (Fig. 1). The puzzle is that the block shot off-center carries rotational kinetic energy in addition to the gravitational potential energy. This leads a majority of the bloggers to claim that the block shot off-center should not go as high as the one shot dead center. Others have claimed that the energy tied up as rotational energy is insignificant and the two blocks should rise to the same height within experimental error.

This note will examine the speculative claims and provide a clear solution to the puzzle. The correct fundamental principle that applies to this collision is the principle of conservation of momentum. The collision is inelastic ("sticky") because the bullet is embedded in the wooden block. The mechanical energy (potential and kinetic) is not conserved in inelastic collisions. Let the mass of the bullet be m and the mass of the block be M. Then conservation of linear momentum in the vertical direction means that the initial linear momentum of the bullet equals the final linear momentum of the bullet-block combo.

$$mV_{\mathrm{i}} = (m+M)\, V_{\mathrm{f}} \qquad (1)$$

$$V_{\mathrm{f}} = [m/(m+M)]\, V_{\mathrm{i}}\,. \qquad (2)$$

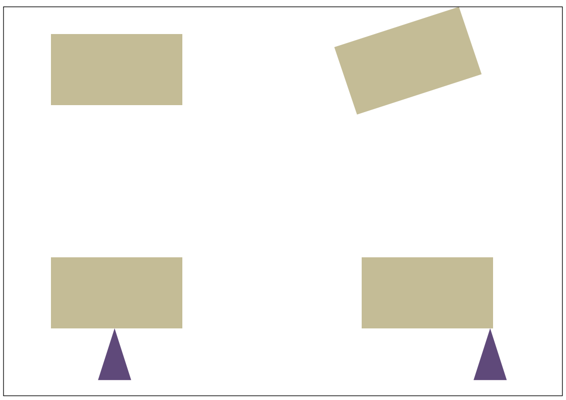

Fig. 1. A bullet hits one block of wood dead center and another off-center.

Thus, the final velocity of the bullet-block combo immediately after the bullet hits the block does not depend at all on whether the block spins or not as it rises. The center of mass of the block hit dead center will rise straight up to the same height as the center of mass of the one hit off-center and spins as it rises.

A misguided conservation law

Those who claim that the spinning block should not rise as high appear to be applying a misguided conservation law. See Ref. 4 for a discussion of many aspects of student thinking regarding this scenario. Evidently, they believe that the total amount of gravitational potential energy and rotational kinetic energy must be conserved. Since the spinning block has some rotational kinetic energy, it must consequently have less gravitational potential energy and must not rise as high as the block that is struck dead center by the bullet and does not spin. The shaky foundation on which this misguided conservation law rests appears to be another misguided conservation law that can be stated in two parts.

(a) The total kinetic energy of the bullet is transferred as the total kinetic energy (gravitational potential energy and rotational kinetic energy) of the block.

(b) Even if (a) is not true,[5] at least the same fraction of the total kinetic energy of the bullet is transferred as the total energy (gravitational potential energy and rotational kinetic energy) of the block.

Debunking misguided conservation law part (a)

Let us first quickly and unceremoniously dispose of misguided conservation law part (a). The bullet loses almost all its kinetic energy either compressing the wood or doing work against friction and generating heat as it gets embedded in the block. *Only a negligible amount of the bullet's initial kinetic energy is transferred as kinetic energy to the block regardless of whether it spins or not.*

This can be proved by noting that the ratio of the translational kinetic energy of the bullet-block combo to the initial kinetic energy of the bullet is

Translational KE of bullet-block combo/ Initial KE of bullet $= ½\,(m+M)\,(V_{\mathrm{f}})^2 \,/\, ½\, m\,(V_{\mathrm{i}})^2$.

Substituting $V_{\mathrm{f}} = [m/(m+M)]\, V_{\mathrm{i}}$ from Eq. (2) and simplifying gives

Translational KE of bullet-block combo/ Initial KE of bullet $= m\,/(m+M)$. (3)

Let's consider a concrete example: A 20-g bullet with a speed of 400 m/s has an initial kinetic energy of 1600 J. In either a dead-center (nonspinning) or off-center (spinning) collision with a 20 cm x 10 cm x 10 cm block of wood having a mass of 1.4 kg, 98.6% of the bullet's initial kinetic energy is not transferred to the block as translational kinetic energy. (These numerical values do not match those in the video exactly, but produce qualitatively similar results.) The bullet-block combo receives only 23 J of the initial 1600 J as translational kinetic energy. This is transformed into gravitational PE as the block rises to a height of only 1.6 m. To put things in perspective, if all the initial KE of the bullet transformed into gravitational PE, the block would rise to a mind-boggling height of 114 m!

Debunking misguided conservation law part (b)

The block that is hit off-center by the bullet spins. Let us apply the conservation of angular momentum to the spinning block. If the block is hit off-center at the edge of the block, the angular momentum of the bullet with respect to the center of mass of the block must equal the angular momentum of the spinning block.

$$m\, d\, V_{\mathrm{i}} = I\omega \qquad (4)$$

Using $d = 0.1$ m and $I = 5.8 \times 10^{-3}$ kg·m^2 for our block, we obtain

$$\omega = 138 \text{ rad/s}.$$

This corresponds to a rotational kinetic energy of the spinning block of

$$\tfrac{1}{2} I\omega^2 = 55 \text{ J}.$$

The spinning block receives a greater fraction of the initial kinetic energy of the bullet in the form of mechanical energy (translational and rotational kinetic energies). It receives the *same translational kinetic energy* as the nonspinning block. This fact accounts for why the two blocks rise to the same height. However, in addition, it also receives some rotational kinetic energy. In fact, in our example, the rotational KE is more than twice the translational KE! So one cannot wiggle out by proclaiming the "approximately true doctrine" that the kinetic energy of rotation is insignificant compared to the translational kinetic energy. What is correct is that either form of kinetic energy is insignificant compared to the initial kinetic energy of the bullet.

In which block does the bullet penetrate deeper?

The above analysis suggests that the spinning block gains more kinetic energy from the bullet than the nonspinning block. The bullet has more of its initial kinetic energy available to penetrate the nonspinning block. The bullet should penetrate the nonspinning block deeper. But is this difference easily measurable? The answer is no. Consider that in our example above, the bullet had a total KE of 1600 J. The KE of the nonspinning block was 23 J. This leaves a whopping 1577 J to drill through the block. In the case of the spinning block, the total KE is 78 J, leaving 1522 J to drill through the block. The 4% percent difference in the length of the bullet hole will be virtually impossible to distinguish. Incidentally, the online video makes some contradictory statements in this regard. At first, the Veritasium investigators poke the holes with a nail and see a big difference in the bullet hole. The spinning block hole appears to be a lot smaller. Then they backtrack and realize that the difference is not so easily measurable. The reason for this discrepancy is that the hole in the spinning block is curved. Poking a nail inside this block will seem to indicate that the hole is very small. An x-ray image is what finally nailed it (pun intended!).

Conclusion

A block of wood will rise to the same height whether the bullet hits it dead center and the block does not spin or hits it off-center and the block spins. This follows from the principle of conservation of linear momentum. The kinetic energy of the bullet-block combo after the collision is only an insignificant fraction of the initial kinetic energy of the bullet. Most of the initial kinetic energy of the bullet is used up in either compressing the wood or doing work against friction and generating heat as it gets lodged in the block. This is true whether the block spins or not. The spinning block does have more total kinetic energy than the one that does not spin. However, both blocks have the same amount of translational kinetic energy. This is consistent with the two blocks rising to the same height.

References

1. The science video blog is Veritasium and the YouTube link is https://www.youtube.com/watch?v=vWVZ6APXM4w.
2. A follow-up Veritasium video where the misguided conservation law is invoked can be found at https://www.youtube.com/watch?v= BLYoyLcdGPc.
3. An interactive video vignette was presented by David Jackson and Priscilla Laws at the AAPT winter 2014 meeting, http://www.compadre.org/psrc/items/detail.cfm?ID=13324.
4. Hunter G. Close and Paula R. L. Heron, "Student understanding of the angular momentum of classical particles," *Am. J. Phys*. **79**, 1068 (Oct. 2011).
5. The ballistic pendulum problem, where a bullet gets lodged in a freely suspended pendulum, which subsequently rises up, has caused grief to countless generations of unsuspecting (read "unprepared") students. Once they start the problem with the misguided conservation law that the total kinetic energy of the bullet must reappear as the kinetic energy of the bullet-block combo, their fate is sealed! This classic gem of physics problems has been around for generations before the bullet-block video made its appearance. A mutant variant of the ballistic pendulum problem rears its ugly head as an initially charged capacitor that is then connected to a similar but uncharged capacitor. A misguided assumption is that the total electrical energy of the first capacitor will simply be redistributed between the two capacitors. The correct conservation law is the conservation of charge.

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