Working With Exponential and Logarithm Functions

The Exponential Function Base e for Continuous Growth or Decay

Given a situation where a population is growing (or decaying) continuously at rate r, or 100r%, per time period for t time periods and the population at time t = 0 is c, the population after t time periods, denoted by P(t), is given by the following rule:

$$P(t) = ce^{rt}$$

Example. India's population in 2008 was estimated to be about 1.148 billion and its annual rate of population growth was estimated to be 1.46%. (a) Assuming, India's growth rate remains constant for the next two decades, what will be India's population in 2028? (b) About when did India's first exceed 1 billion?

Example. The carbon atom has a radioactive isotope called C-14 that occurs naturally in small quantities. This isotope combines with oxygen to form CO_2 in the atmosphere, which is absorbed by plants and animals during their lives. When a living thing dies, it no longer absorbs C-14. If organic material becomes buried and is kept form absorbing fresh C-14 from air or water, then the concentration of C-14 in the organic material will decrease over time due to the process of radioactive decay. The half-life of the decay process is known to be 5715 years. The radioactivity of living material due to the presence of C-14 is estimated to be 14.6 atomic disintegrations per minute per gram of carbon, as measured by a Geiger counter.

If we let R(t) = the radio activity remaining in the sample t years after being buried, then

$R(t) = 14.6e^{rt}$ where r denotes the rate of decay.

We can determine the decay rate r by using the fact that C-14's half life is 5715 years.

$$R(5715) = 7.3 = 14.6e^{r(5715)}$$

Solving for r,

$$0.5 = e^{5715r} \rightarrow \ln(0.5) = 5715r \rightarrow r \approx -0.0001213$$

Hence,

$$R(t) \approx 14.6e^{-0.0001213t}.$$

Suppose a bone fragment found in a burial mound has a radioactivity count of 6.32 counts per minute per gram of carbon. How old is the bone fragment?

Rules for Calculating with Logarithms

Rule	*For Logarithms*	*For Exponents*
Product Rule	**$Ln(xy) = Ln(x) + Ln(y)$**	**$e^x e^y = e^{x + y}$**
Quotient Rule	**$Ln(x/y) = Ln(x) – Ln(y)$**	**$e^x/e^y = e^{x - y}$**
Power Rule	**$Ln(x^y) = yLn(x)$**	**$(e^x)^y = e^{xy}$**

Make use of the rules for exponents and logarithms in solving the following equations.

1. **$100 = 50e^{0.07x}$**
2. **$800 = 2000e^{20r}$**
3. **$Ln(2x) = 4$**
4. **$Ln(5x – 3) = 10$**

Solve for x:

4. **$y = 1000e^{0.25x}$**
6. **$Ln(3x + 8) = 12$**