Some Properties of Quadratic Functions

We have seen that for the general quadratic function f(x) = ax^2 + bx + c where a $\neq$ 0, the graph of f is a _______________ with vertex at _____________________________. The parabola opens up if ___________ and opens down if ___________. The graph's intercept on the vertical axis is at _____________. The intercepts on the horizontal axis, if any exist, will correspond to roots of the function. That is, at those values for x such that f(x) = 0. (How can it occur that there are no intercepts on the horizontal axis?)

Let's find all values of x such that f(x) = 0, if any exist.

0 = ax^2 + bx + c

$$0 = a(x - \frac{-b}{2a})^2 + \frac{4ac - b^2}{4a}$$ **(How do we know this?)**

$$\frac{b^2 - 4ac}{4a^2} = (x + \frac{b}{2a})^2$$

$$(x + \frac{b}{2a}) = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$ **(Why do we have the "$\pm$" symbol?)**

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

So, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$**. (Under what conditions do solutions actually exist?)**