Exponential Growth

An initial amount of A grows at rate r per year compounded n times per year. The amount after t years, denoted by V(t) is given by

$V(t) = A(1+\frac{r}{n})^{nt}$ Which is equivalent to

$$V(t) = A\left[1+\frac{1}{\left(\frac{n}{r}\right)}\right]^{\left(\frac{n}{r}\right)rt}$$

We want to consider what happens to V(t) as the number of compounding periods increases without bound. That is, becomes infinite.

Suppose we replace (n/r) with x.

$$V(t) = A\left\{\left[1+\frac{1}{x}\right]^{x}\right\}^{rt}.$$

We can consider what happens to V(t) as n becomes large by considering what happens as x becomes large. Complete the table below.

x	$\left[1+\frac{1}{x}\right]^{x}$
10	
100	
1000	
10000	
100000	
1000000	

So, for very large values of x, $\left[1+\frac{1}{x}\right]^{x}$ is about 2.7182.

Hence, for very large values of n,

$$V(t) \approx A\{2.7182\}^{rt}$$

Actually, for arbitrarily large values of n, we have

(*) $V(t) \approx Ae^{rt}$

e ≈ 2.7182 and e is a number that cannot be expressed exactly as a decimal.

Continuous exponential growth (decay) is modeled by equation (*).