## Math 100 More Class Work on Functions and Functional Notation

Suppose the variables " $x$ " and " $y$ " represent quantities that are in a functional relationship. We often use the symbolism

$$
\mathbf{y}=f(\mathbf{x})
$$

to indicate that $y$ is a function of $x$. We read this equation as " $y$ equals $f$ of $x$." The symbol " $f(x)$ " is another symbol for the variable $\mathbf{y}$. Our independent variable is $\mathbf{x}$ and y is the dependent variable because its value depends on the value of $x$. The set of values that the independent variable can assume is called the domain of the function; the set of values that the dependent variable can assume is called the range of the function.

Example 1: Suppose that an object is dropped from a height of 500 feet. Between the time the ball is released and the time it hits the ground, the distance the object has fallen depends on the time elapsed since that ball was released.

## Suppose

$t$ = the number of seconds since the ball was released, and $d(t)=$ the distance in feet the ball has fallen in $t$ seconds.

The distance the ball has fallen in $\mathbf{t}$ seconds can be described by the function

$$
\mathbf{d}(\mathbf{t})=16 \mathbf{t}^{2}
$$

We denote the distance (in feet) fallen in 2 seconds by $d(2)$. In this case $d(2)=64$; so the ball falls 64 feet in 2 seconds.

Calculate and interpret the meaning of $d(3)$ and $d(5)$.

Complete the table and sketch a graph of this function.

t
What is a reasonable domain for this function?

What is a reasonable range for this function?

Example 2: Suppose a business purchases a new computer for $\$ 4000$. The value of the computer drops or depreciates by $\mathbf{\$ 5 0 0}$ per year following the purchase.

Let $t=$ the number of years since the computer was purchased, and $\mathbf{V}(\mathbf{t})=$ the depreciated value of the computer $\mathbf{t}$ years after being purchased.
a. Write a formula for the functional relationship between $V(t)$ and $t$. Specify a meaningful domain and range for the function.
b. Calculate and interpret the meaning of V(4)
c. Graph the function and interpret the meaning of two intercepts and the slope.


| $\Delta \mathbf{t}$ | $\mathbf{t}$ | $\mathbf{V}(\mathbf{t})$ | $\Delta \mathbf{V}(\mathbf{t})$ | $\Delta \mathbf{V}(\mathbf{t}) / \Delta \mathbf{t}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{0}$ |  |  |  |
|  | 2 |  |  |  |
|  | 4 |  |  |  |
|  | 6 |  |  |  |

Example 3. Suppose a population of bacteria in a culture is growing at the rate of $20 \%$ per day. If we start with a population of 2000 bacteria, how might we express the relationship between the number of bacteria in the culture and the number of days elapsed?

Let $P(t)=$ the number of bacteria (in thousands) present in the culture after $\mathbf{t}$ days.

| t | $\mathrm{P}(\mathrm{t})$ | $\Delta \mathrm{P}(\mathrm{t})$ | $\Delta \Delta \mathrm{P}(\mathrm{t})$ | $\mathrm{P}(\mathrm{t}) / \mathrm{P}(\mathrm{t}-1)$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 2.000 |  |  |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 | 3.456 |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |



Complete the table and graph and try to find a rule relating $P(t)$ and $t$.

Example 4. Consider the function defined by the rule $(\boldsymbol{p})=\frac{100}{p}$. Complete the table and graph.

| $p$ | $D(p)$ | $\Delta D(p)$ | $\Delta \Delta D(p)$ | $D(p) / D(p-1)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 100.000 |  |  |  |
| 2 | 50.000 | -50.000 |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |



