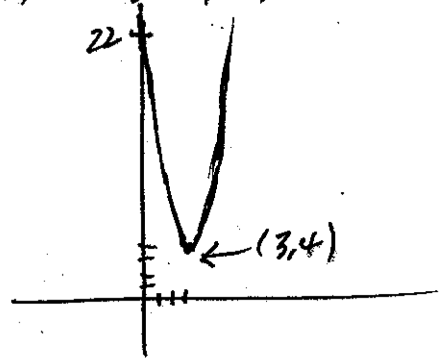
Sample Exercises

On Monday we considered the general quadratic form. $y=ax^2+bx+c$
and asked "what can we tell about the graph of the function by looking at the values of a, b, and c?"

When we looked at the form $y=a(x-h)^2+k$ we concluded the graph of $y=a(x-h)^2+k$ is a parabola that opens up if $a>0$ and down if $a<0$. The vertex is at the point (h,k).

So the graph of $y=2(x-3)^2+4$ is a parabola that opens up with vertex at $(3,4)$



Expanding $y=a(x-h)^2+k$ we determined that

$$y = \underset{\substack{\uparrow \\ a}}{a}x^2 + (\underset{\substack{\uparrow \\ b}}{-2ah})x + (\underset{\substack{\uparrow \\ c}}{ah^2+k})$$

Solving $-2ah=b$ for h we get $h=\dfrac{-b}{2a}$.

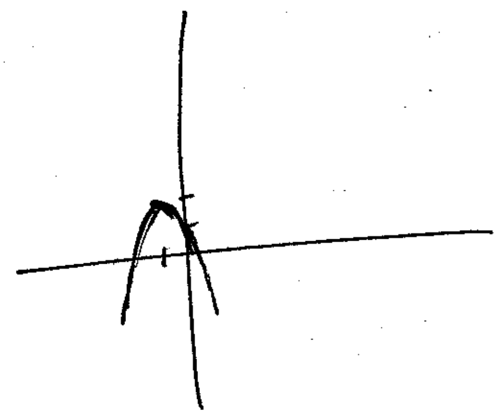
Therefore $y=ax^2+bx+c$ has a graph that is a parabola with vertex at $x=\dfrac{-b}{2a}$.

Page 544
#9

$y = -x^2 - 2x + 1$

$y = -1x^2 + (-2)x + 1$
$\quad\;\; \uparrow \qquad\quad \uparrow$
$\quad\;\; a \qquad\quad\; b$

The graph is a parabola that opens down $(a = -1 < 0)$
with vertex where $x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)} = -1.$
If $x = -1$, $y = 2$.
The y-intercept is at 1.



page 545
#11

Parabola opens down
vertex at approximately (7,23). so
$y = a(x-7)^2 + 23$
Since y-intercept is about at 10. We seek a value
for a so that
$10 = a(0-7)^2 + 23$
Hence, $a = \frac{-13}{49}$
Therefore, our equation is about $\boxed{y = \frac{-13}{49}(x-7)^2 + 23}$

Page 544
#21

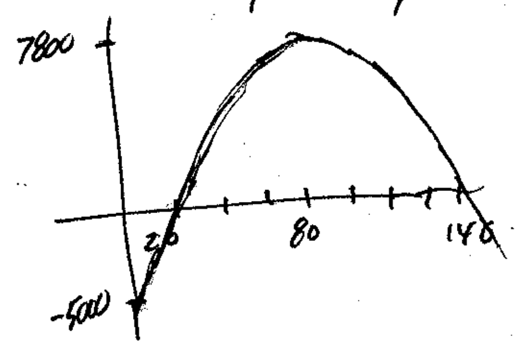
The graph of $\Pi(Q) = -2Q^2 + 320Q - 5000$ is a parabola that opens down with vertex where $Q = \frac{-320}{2(-2)} = 80$

a) If $Q = 80$, $\Pi(80) = 7800$

So, the vertex is at $(80, 7800)$

The maximum profit of \$7800 is achieved when the quantity sold is 80.

b)



Page 546
#23

$C(x)$ = ave cost of production when x units are produced (\$)

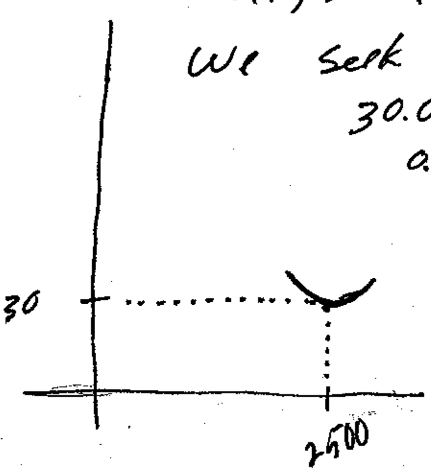
Our vertex is at $(2500, 30)$ So,

$C(x) = a(x-2500)^2 + 30$

We seek a so that when $x = 2501$, $C(x) = 30.01$

$30.01 = a(2501-2500)^2 + 30$

$0.01 = a$.



Hence our equation is

$$\boxed{\begin{gathered} C(x) = 0.01(x-2500)^2 + 30 \\ \text{or} \\ C(x) = 0.01x^2 - 50x + 62530 \end{gathered}}$$