A Binomial Random Variable Revisited

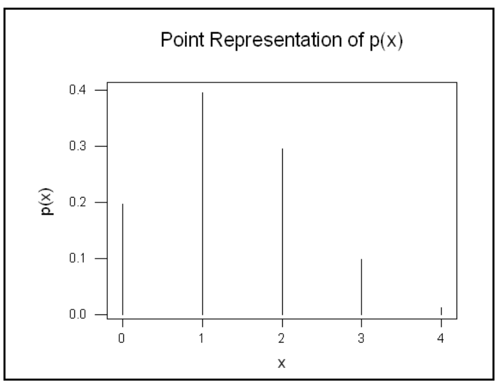
Recall the experiment of last week in which a fair die is tossed four times and we classified the result each toss as a success (S) if one or two dots appeared on the top face and as a failure (F) otherwise. We were interested in the number of successes (S's) in four tosses. The sample space for this experiment is shown below.

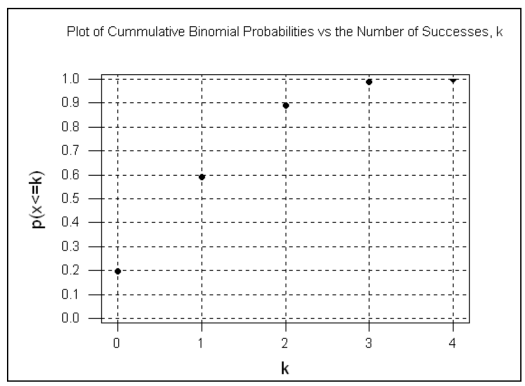
{SSSS, FSSS, SFSS, SSFS, SSSF, FFSS, FSFS, FSSF, SFFS, SFSF, SSFF, SFFF, FSFF, FFSF, FFFS, FFFF}

The random variable x associated each sample point with its number of successes, and we assigned a probability p(x) to each value of the random variable. The function $x \rightarrow p(x)$ is the probability distribution for this experiment.

Event	x	p(x)	$\approx p$(x)
SSSS	4	1/81	0.0125
FSSS, SFSS, SSFS, SSSF	3	8/81	0.0988
FFSS, FSFS, FSSF, SFFS, SFSF, SSFF	2	24/81	0.2963
SFFF, FSFF, FFSF, FFFS	1	32/81	0.3951
FFFF	0	16/81	0.1975

Here are graphs of this probability distribution for x and its cumulative probabilities.





We calculated E(x) the expected value, or mean (μ), and standard deviation (σ) of the random variable x and interpreted the results. Recall that $\mu = E(x) = \sum x\, p(x)$ and $\sigma^2 = E[(x-\mu)^2] = \sum (x-\mu)^2\, p(x)$.

x	p(x)	xp(x)	x - μ	$(x-\mu)^2$	$(x-\mu)^2 p(x)$
0	0.1975	0	-1.334	1.780	0.352
1	0.3951	0.3951	-0.334	0.112	0.044
2	0.2963	0.5926	0.666	0.444	0.131
3	0.0988	0.2963	1.666	2.776	0.274
4	0.0124	0.0494	2.666	7.108	0.088

$\mu = E(x) = \sum x\, p(x) = 1.334$; $\sigma^2 = E[(x-\mu)^2] = \sum (x-\mu)^2\, p(x) = 0.889$; $\sigma = 0.943$

In experiments like this one where there are two possible results (success or failure) for each independent and identical trial and the probabilities remain the same for each trial and we observe the number x of successes, we call x a ***binomial random variable***. In this example we had a random variable with 4 trials and the probability of success on any trial is 1/3.

TI Output Values: $\bar{x} = 1.3333$; $\sigma = 0.9428$

Finding μ, σ^2, and σ for a Binomial Random Variable

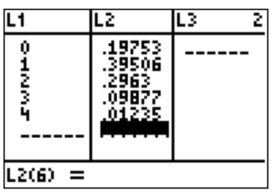
We will not need to use the summation definitions we have been employing to calculate μ, σ^2, and σ for a binomial random variable.

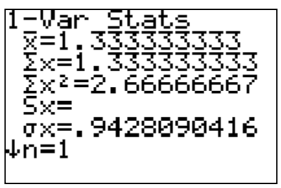
> For a binomial random variable x = the number of successes where p = the probability of success on a single trial, and n = the number of trials we have the following rules:
>
> Mean: $\mu = np$
>
> Variance: $\sigma^2 = np(1- p)$
>
> Standard Deviation: $\sigma = \sqrt{(np(1-p)}$

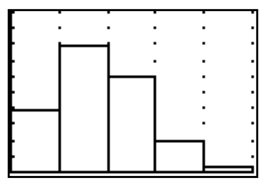
We verified the above rules for the random variable x of this example.

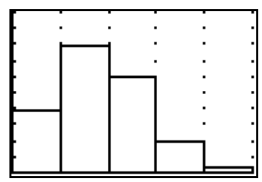
Mean: $\mu = np = 4(1/3) = 1.333\ldots$
Variance: $\sigma^2 = np(1- p) = 4(1/3)(2/3) = 0.888\ldots$
Standard Deviation: $\sigma = \sqrt{(np(1-p)} = \sqrt{\sigma^2} = \sqrt{0.889} \approx 0.943$

We can obtain the following graphing calculator displays for this example where L_1 contains the values of the random variable x and L_2 contains the associated probabilities p(X = x):





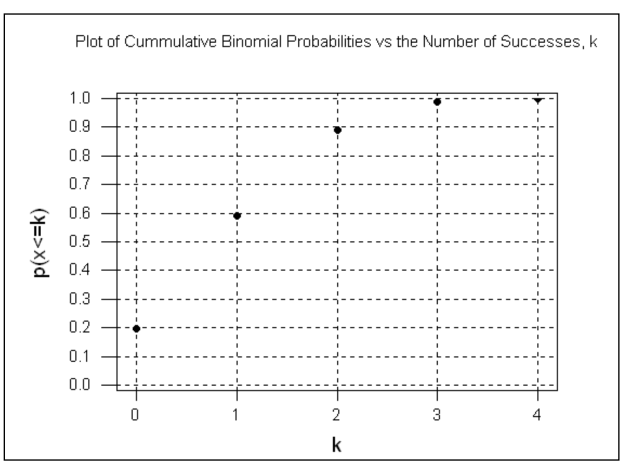


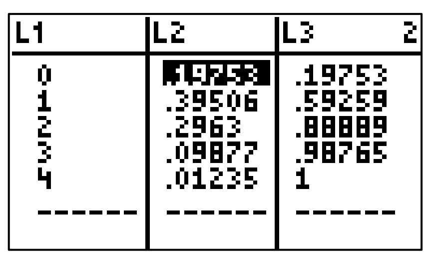


P(X≤ 2) is shaded

Using the TI-85/TI-84 to calculate the mean and standard deviation is demonstrated in pp. 178-179 in our text. Try to generate the output shown above.

We can examine the cumulative binomial probabilities in the graph below on the left. We can also see those cumulative probabilities in the list L_3 is the calculator display below on the right.





L_1 lists the values x can assume.
L_2 gives the associated values of P(x).
L_3 gives the values of $P(X \le x)$

Determine each of the following probabilities by two or three different methods.

$P(x \leq 2) =$ ______________ $P(x = 2) =$ _____________

$P(x \leq 3) =$ ______________ $P(0.39 < x < 2.28) =$ ____________

If you have a TI-83 or TI-84, try the following sequences:

a) Press 2nd, VARS, under DISTR, press down arrow until binompdf(is found, ENTER. After binompdf(type 4, 1/3, 2) and then press ENTER. (You should find P(x = 2)).

b) Press 2nd, VARS, under DISTR, press down arrow until binomcdf(is found, ENTER. After binomcdf(type 4, 1/3, 2) and then press ENTER. (You should find P(x ≤ 2)).

See pp. 193-194 in our text for instructions on calculating binomial probabilities.

Example of Another Binomial Distribution

A national poll conducted by *The New York Times* (May 7, 2000) revealed that 80% of Americans believe that after one dies, some part of him or her lives on, either in a next life on earth or in heaven. Assuming the polls' results are correct, consider a random sample of 10 Americans and count x, the number who believe in life after death.

a. Explain why x is (approximately) a binomial random variable.

b. Use the Table 4.4 below to help you determine each of the following probabilities. In this case, $n = 10$ and $p = 0.80$.

$P(x = 3) =$ ________ $P(x \leq 7) =$ ________ $P(x > 4) =$ ________

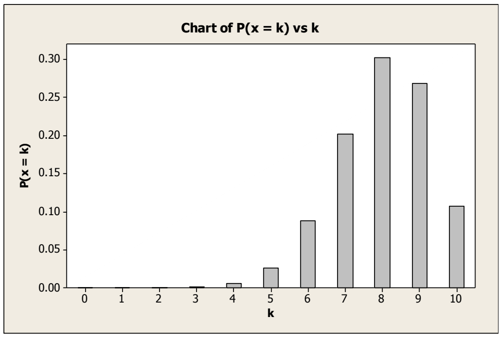
This table gives **cumulative binomial probabilities** $P(x \leq k)$.

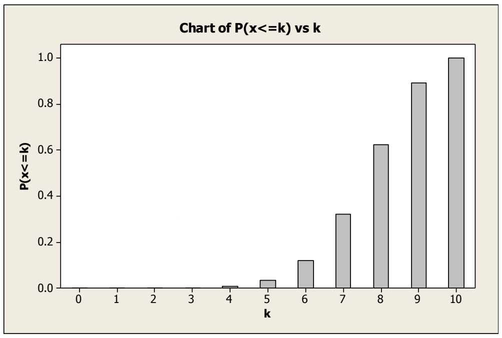
TABLE 4.4 Reproduction of Part of Table II of Appendix A: Binomial Probabilities for $n = 10$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.904	.599	.349	.107	.028	.006	.001	.000	.000	.000	.000	.000	.000
1	.996	.914	.736	.376	.149	.046	.011	.002	.000	.000	.000	.000	.000
2	1.000	.988	.930	.678	.383	.167	.055	.012	.002	.000	.000	.000	.000
3	1.000	.999	.987	.879	.650	.382	.172	.055	.011	.001	.000	.000	.000
4	1.000	1.000	.998	.967	.850	.633	.377	.166	.047	.006	.000	.000	.000
5	1.000	1.000	1.000	.994	.953	.834	.623	.367	.150	.033	.002	.000	.000
6	1.000	1.000	1.000	.999	.989	.945	.828	.618	.350	.121	.013	.001	.000
7	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.322	.070	.012	.000
8	1.000	1.000	1.000	1.000	1.000	.988	.989	.954	.851	.624	.264	.086	.004
9	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.893	.651	.401	.096

Here is some Minitab output for this example. Compare to entries in Table 4.4.

k	prob $x = k$	cumulative prob $x \le k$
0	0.000000	0.00000
1	0.000004	0.00000
2	0.000074	0.00008
3	0.000786	0.00086
4	0.005505	0.00637
5	0.026424	0.03279
6	0.088080	0.12087
7	0.201327	0.32220
8	0.301990	0.62419
9	0.268435	0.89263
10	0.107374	1.00000





According to the assumptions of this example, respond to each of the following questions.

a. In a random sample of 10 Americans, what is the probability that at least 5 of them believe in life after death?

b. In a random sample of 10 Americans, what is the probability that exactly 5 of them believe in life after death?

c. In a random sample of 10 Americans, what is the probability that more than 8 of them believe in life after death?

d. In a random sample of 10 Americans, what is the probability that at most 8 of them believe in life after death?

See pages 249 and the top portion of page 250 for instructions on using MINITAB to obtain binomial probabilities. Try to produce output like that shown above.