Sampling Distributions Revisited – Applying the CLT

Parameters and Corresponding Statistics			
	Population Parameter	**Sample Statistic**	**Sampling Distribution Statistic**
Mean	μ	$\bar{x}$	$\mu_{\bar{x}} = \mu$
Variance	σ^2	s^2	
Standard Deviation	σ	s	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

The Central Limit Theorem (CLT)

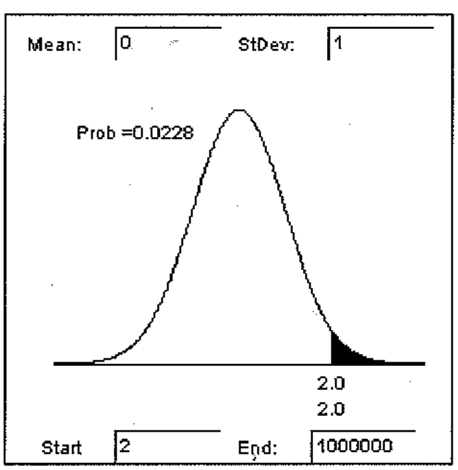
Given a distribution with a mean μ and variance σ^2, the sampling distribution of the mean approaches a normal distribution with a mean μ and variance σ^2/n as *n*, the sample size, increases. Regardless of the shape of the original distribution, the sampling distribution of the mean approaches a normal distribution. Usually a normal distribution is approached very quickly as *n* increases. *n* is the sample size for each mean and not the number of samples. In a sampling distribution the number of samples is assumed to be infinite. The sample size is the number of scores in each sample; it is the number of measurements that go into the computation of each mean. (This definition is adapted from David Lane's "Hyperstat Online Statistics Textbook" at http://davidmlane.com/hyperstat/index.html.)

Using the CLT to Find a Probability

Example: Suppose a random sample of size *n* = 49 observations is selected from a population with mean μ = 90 and standard deviation σ = 7. The population is not extremely skewed. Find the probability that $\bar{x}$ will be greater than 92.

By the CLT, the sampling distribution for $\bar{x}$ is normal with $\mu_{\bar{x}} = \mu = 90$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 1$. So, for $\bar{x}$ = 92 we have $z = (\bar{x} - \mu_{\bar{x}})/\sigma_{\bar{x}}$ = (92 – 90)/1 = 2. Consulting Table III we find the tail area corresponding to the probability that $\bar{x}$ exceeds 92 is

$$P(\bar{x} > 92) = P(z > 2) = 0.5 - 0.4772 = 0.0228.$$



A Practical Application

A company claims that students who paid its tutors to help them improve their SAT scores had a mean score change of +40 points, with a standard deviation of 32 points. In a random sample of 64 students who paid the company for a private tutor to help them improve their SAT scores, it was found that the mean score change for students in the sample was +31 points. Should we believe the company's claims?

If the company's claim is true, the probability that the sample group's mean score change was +31 or less is denoted by $P(\bar{x} \leq 31)$. By the CLT, the sampling distribution for $\bar{x}$ is $N(40,4)$, and $P(\bar{x} \leq 31) \approx 0.0122$. So, if the company's claim is true, the probability that our sample group will have a sample mean score change of 31 points or less is 0.0122.