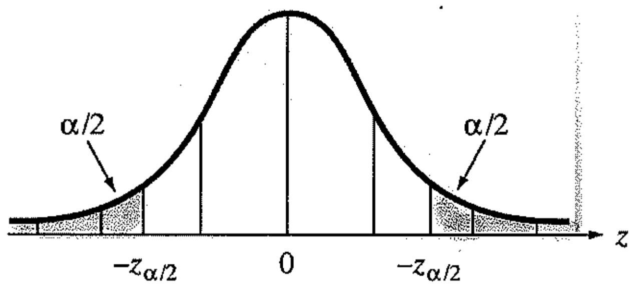
Finding a Large Sample (n ≥ 30) Confidence Interval (Summary Review)

The ***confidence coefficient*** is equal to 1 - α , and is split between the two tails of the distribution.



The large sample 100(1 - α)% ***confidence interval* for μ** is

$$\bar{x} \pm z_{\alpha/2}\sigma_{\bar{x}} = \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \approx \bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

where $\bar{x}$ is the sample mean, s is the sample standard deviation, n is the number of measurements in the sample, and $z_{\alpha/2}$ is the value with an area of $\alpha/2$ to its right. The sample used is required to be random.

To calculate the 90% confidence interval for μ we use the following form:

$$\bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = \bar{x} \pm z_{0.05}\left(\frac{s}{\sqrt{n}}\right) = \bar{x} \pm 1.645\left(\frac{s}{\sqrt{n}}\right)$$

To calculate the 95% confidence interval for μ we use the following form:

$$\bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = \bar{x} \pm z_{0.025}\left(\frac{s}{\sqrt{n}}\right) = \bar{x} \pm 1.96\left(\frac{s}{\sqrt{n}}\right)$$

To calculate the 99% confidence interval for μ we use the following form:

$$\bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = \bar{x} \pm z_{0.005}\left(\frac{s}{\sqrt{n}}\right) = \bar{x} \pm 2.575\left(\frac{s}{\sqrt{n}}\right)$$

Example: A random sample of 46 observations produced a mean of $\bar{x} = 19.3$ and $s = 11.9$. Calculate a 95% confidence interval for μ.

$$\bar{x} \pm 1.96\left(\frac{s}{\sqrt{n}}\right) = 19.3 \pm 1.96\left(\frac{11.9}{\sqrt{46}}\right) \approx 19.3 \pm \underline{\qquad\qquad}.$$

So, our 95% confidence interval is ______________________________.

Finding a Small Sample (n < 30) Confidence Interval

With small samples we have two immediate problems. First, the shape of the sampling distribution of the sample mean now depends on the shape of the population being sampled. However, the sampling distribution of $\bar{x}$ is approximately normal even for small samples if the sampled population is approximately normal. Second, the sampled population's standard deviation σ is almost always unknown and in this small sample case the sample's standard deviation *s* may provide a poor approximation for σ.

So, rather than using the standard normal statistic *z*, $z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$,

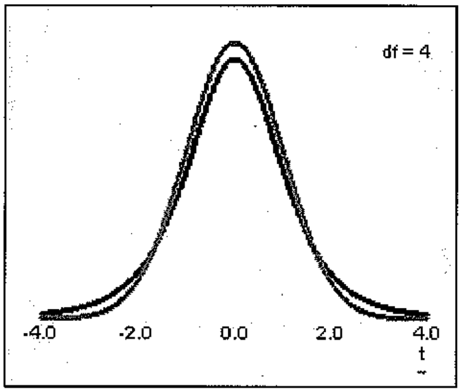
we define and use the following ***t*-statistic**. $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$.

The t-statistic is more variable than the z-statistic and the amount of variability depends on the sample size *n*. We express this dependence by saying that the t-statistic has (*n* - 1) **degrees of freedom.**

The t-distribution has a shape very much like that of the z-distribution. In the figure below the normal z-distribution is the one with the higher peak. The graph of the t-distribution with 4 degrees of freedom is slightly below the graph of the normal z-distribution over the interval (-1, 1). Values of t that are used in small-sample confidence intervals are given in Table IV of Appendix A.

In cases where we can assume that the population has a relative frequency distribution that is approximately normal and we have a randomly selected sample from the target population, the small-sample confidence interval for μ is

$\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$ where $t_{\alpha/2}$ is based on (n – 1) degrees of freedom (df).



An applet allowing one to compare t and z for various degrees of freedom can be found at http://www.uvm.edu/~dhowell/SeeingStatisticsApplets/TvsZ.html

Example. Compute a 99% confidence interval for the mean, using the following 15 data values.
1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29

$n =$ _______; sample mean $\bar{x} =$ _______; sample standard deviation $s =$ _______;

$\alpha =$ _______; $\alpha/2 =$ _______; $t_{\alpha/2} =$ _______; df = _______; $t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) =$ _______;

$\bar{x} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) =$ _______; $\bar{x} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) =$ _______.

So, our 99% confidence interval is ________________________. (See page 272.)