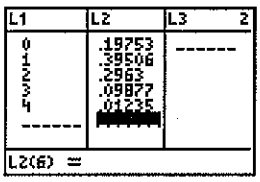
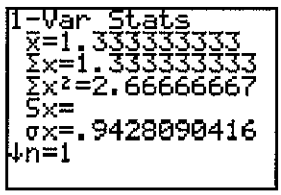
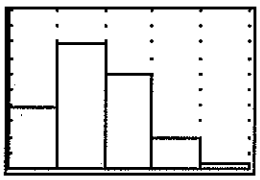
The Binomial Distribution Continued

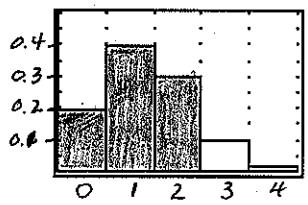
Return to the experiment in which a fair die is tossed four times and we classify the result each toss as a success (S) if one or two dots appear on the top face and as a failure (F) otherwise. We are interested in the x the number of successes (S's) in four tosses.

We obtained the following graphing calculator displays for this example where L_1 contains the values of the random variable x and L_2 contains the associated probabilities p(X = x):



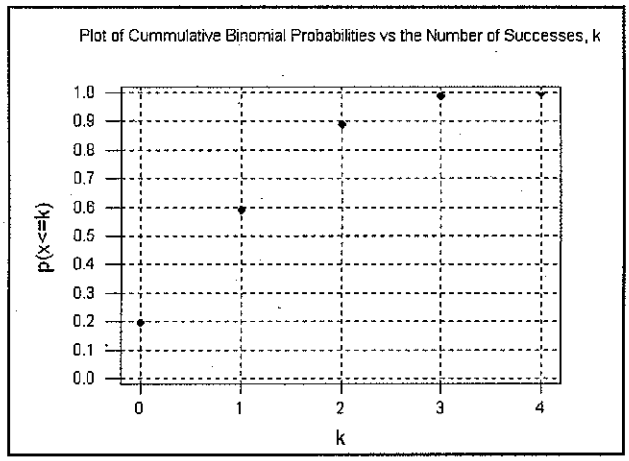






P(x ≤ 2) is shaded

We also examined the cumulative binomial probabilities in the graph below on the left. We can also see those cumulative probabilities in the list L_3 is the calculator display below on the right.



L1	L2	L3 2
0	.19753	.19753
1	.39506	.59259
2	.2963	.88889
3	.09877	.98765
4	.01235	1
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L2(1)=.1975308641...

L_1 lists the values x can assume.
L_2 gives the associated values of P(x).
L_3 gives the values of P(X ≤ x)

Determine each of the following probabilities by two or three different methods.

P(x ≤ 2) = _____ P(x = 2) = _____

P(x ≤ 3) = _____ P(0.39 < x < 2.28) = _____

If you have a TI-83 or TI-84, try the following sequences:

a) Press 2nd, VARS, under DISTR, press down arrow until binompdf(is found, ENTER. After binompdf(type 4, 1/3, 2) and then press ENTER. (You should find P(x = 2)).
b) Press 2nd, VARS, under DISTR, press down arrow until binomcdf(is found, ENTER. After binomcdf(type 4, 1/3, 2) and then press ENTER. (You should find P(x ≤ 2)).

The Binomial Probability Distribution Defined

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad (x = 0, 1, 2, \ldots, n)$$

where

p = probability of success on a single trial
q = 1- p
n = number of trials
x = number of successes in n trials

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

Example of Another Binomial Distribution

A national poll conducted by *The New York Times* (May 7, 2000) revealed that 80% of Americans believe that after you die, some part of you lives on, either in a next life on earth or in heaven. Assuming the polls' results are correct, consider a random sample of 10 Americans and count x, the number who believe in life after death.

a. Explain why x is (approximately) a binomial random variable.

b. Give the probability distribution for x, $p(x)$, as a formula.

c. Use the Table 4.4 below to help you determine each of the following probabilities.

$P(x = 3)$ = _____ $P(x \leq 7)$ = _____ $P(x > 4)$ = _____

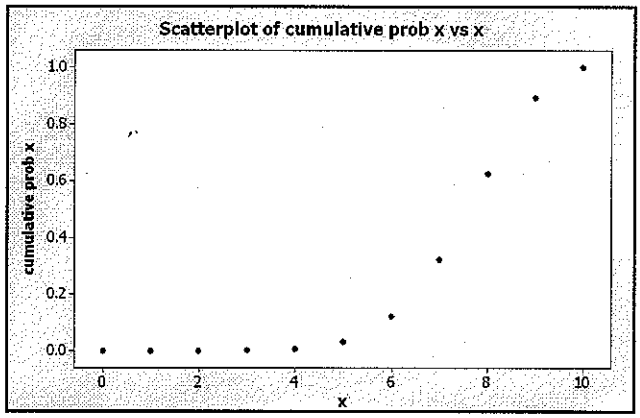
This table gives **cumulative binomial probabilities** $P(x \leq k)$.

TABLE 4.4 **Reproduction of Part of Table II of Appendix A: Binomial Probabilities for n = 10**

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.904	.599	.349	.107	.028	.006	.001	.000	.000	.000	.000	.000	.000
1	.996	.914	.736	.376	.149	.046	.011	.002	.000	.000	.000	.000	.000
2	1.000	.988	.930	.678	.383	.167	.055	.012	.002	.000	.000	.000	.000
3	1.000	.999	.987	.879	.650	.382	.172	.055	.011	.001	.000	.000	.000
4	1.000	1.000	.998	.967	.850	.633	.377	.166	.047	.006	.000	.000	.000
5	1.000	1.000	1.000	.994	.953	.834	.623	.367	.150	.033	.002	.000	.000
6	1.000	1.000	1.000	.999	.989	.945	.828	.618	.350	.121	.013	.001	.000
7	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.322	.070	.012	.000
8	1.000	1.000	1.000	1.000	1.000	.988	.989	.954	.851	.624	.264	.086	.004
9	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.893	.651	.401	.096

Here is some Minitab output for this example. Compare to entries in Table 4.4.

k	prob x = k	cumulative prob x ≤ k
0	0.000000	0.00000
1	0.000004	0.00000
2	0.000074	0.00008
3	0.000786	0.00086
4	0.005505	0.00637
5	0.026424	0.03279
6	0.088080	0.12087
7	0.201327	0.32220
8	0.301990	0.62419
9	0.268435	0.89263
10	0.107374	1.00000



According to the assumptions of this example, respond to each of the following questions.

a. In a random sample of 10 Americans, what is the probability that at least 5 of them believe in life after death?

b. In a random sample of 10 Americans, what is the probability that exactly 5 of them believe in life after death?

c. In a random sample of 10 Americans, what is the probability that more than 8 of them believe in life after death?

d. In a random sample of 10 Americans, what is the probability that at most 8 of them believe in life after death?