Math 155 Test of Hypothesis Group Work

Example 1. A claim has been made the mean height of students at SU is 69 inches. We are interested in determining whether we can accept that claim as true. Assume we have taken a random sample of 64 students from the population of students at SU and found a sample mean of 68 with a standard deviation of 3. Conduct a two tailed test of the claim at the 0.05 significance level.

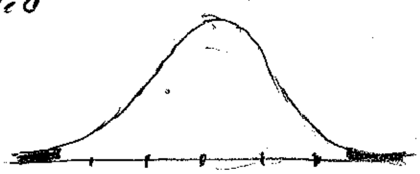
Test Setup

Define *Ho*: $\mu = 69$

Define *Ha*: $\mu \neq 69$

What kind of test (left, right, two-tailed)? Two-tailed

Select α: $\alpha = 0.05$



Conduct Test *(Illustrate with a sketch)*

Determine $\bar{x}$: $\bar{x} = 68$

Determine *s*: $s = 3$

Calculate Test Statistic *z*: $z = \frac{\bar{x} - 69}{\sigma_{\bar{x}}} = \frac{\bar{x} - 69}{3/\sqrt{64}} = \frac{68 - 69}{3/\sqrt{64}} \approx -2.667$

Determine Rejection Region: $z < -1.96$ or $z > 1.96$ $\left(z_{\alpha/2} = z_{0.025} = 1.96\right)$

Determine the *p*-value: p-value = 0.008

Is z in the rejection region? Is the p=value less than α? z lies in the rejection region and p-value = 0.008 < $\alpha = 0.05$

Conclusion

Is *Ho* rejected? Yes, we reject H_0.

Which type of error is possible? A Type I error is possible with probability 0.05

State a conclusion in written form in the context of the experiment.

We have sufficent evidence to conclude, at the $\alpha = 0.05$ level, that the mean height of SU students differs from 69 inches.

Example 2. A claim has been made the mean height of students at SU is 69 inches. We are interested in determining whether we can accept that claim as true. Assume we have taken a random sample of 10 students from the population of students at SU and found a sample mean of 68 with a standard deviation of 3. Conduct a two tailed test of the claim at the 0.05 significance level.

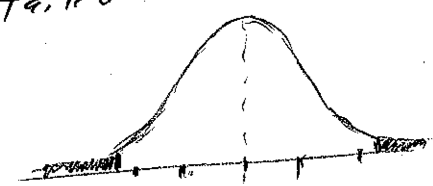
Test Setup

Define *Ho*: $\mu = 69$

Define *Ha*: $\mu \neq 69$

What kind of test (left, right, two-tailed)? two-tailed

Select α: $\alpha = 0.05$



Conduct Test *(Illustrate with a sketch)*

Determine $\bar{x}$: 68

Determine *s*: 3

Calculate Test Statistic *t*: $t = \frac{\bar{x} - 69}{3/\sqrt{n}} \approx \frac{68-69}{3/\sqrt{10}} \approx -1.054$

Determine Rejection Region: $t < -t_{\alpha/2} = -t_{0.025} \approx -2.262$ or $t > 2.262$

Determine the *p*-value: p-value ≈ 0.3193

Is *t* in the rejection region? Is the p=value less than α? t is not in the rejection region.
p-value $\approx 0.319 > \alpha = 0.05$

Conclusion

Is *Ho* rejected? No.

Which type of error is possible? A type II error is possible with unknown probability β.

State a conclusion in written form in the context of the experiment.

We have insufficient evidence to reject the premise that the mean height of SU students is 69 inches.

Example 3. Starbucks claims their coffee contains on average at most 300 mg of caffeine. We wish to test this claim and on seven consecutive days, we buy and test a cup of their coffee for caffeine level. The test results are: 564 310 398 300 307 259 303. (The underlying population is assumed to be normal.)

Parametric T-Test

Test Setup

Ho: $\mu = 300$

Ha: $\mu > 300$

What kind of test (left, right, or two tailed)? right tail test

Select α: $\alpha = 0.05$

Conduct Test

Determine $\bar{x}$: $\bar{x} \approx 348.7$

Determine *s*: 103.7

Calculate *t*: $t = \dfrac{348.7 - 300}{103.7/\sqrt{7}} \approx 1.243$

Determine the *p*-value: p-value = 0.130   Rejection region: $t > 1.943$ with 6 df.

Compare the *p*-value with α: p-value $= 0.13 > \alpha = 0.05$
(t is not in the rejection region)

Test Conclusion

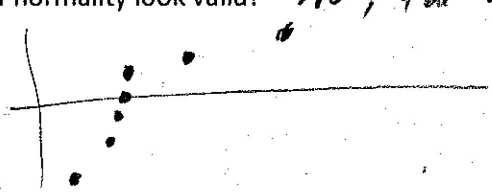
Is *Ho* rejected? No

Which type error is possible? A type II error is possible with unknown probability β.

State a conclusion in written form in the context of the experiment.

We cannot conclude that the mean caffeine level exceeds 300 mg per cup.

Does our assumption of normality look valid? No, the normal probability plot looks like



. So, we probably should not have used this test.

Example 4. Starbucks claims their coffee contains on average at most 300 mg of caffeine. We wish to test this claim and on seven consecutive days, we buy and test a cup of their coffee for caffeine level. The test results are: 564 310 398 300 307 259 303

Non-Parametric Sign Test

Test Setup (Median test)

Ho: $\eta = 300$

Ha: $\eta > 300$

What kind of test (left, right, or two tailed)? right tailed

Select α: $\alpha = 0.05$

Conduct Test

Determine S: $S = 5$ (There is one value (300) that equals the claim)

Determine the *p*-value: $P(X \geq 5) = 1 - P(X \leq 4) = 1 - \text{binomcdf}(6, 0.5, 4) \approx 0.109$

Compare the *p*-value with α: $p\text{-value} > \alpha$

Test Conclusion

Is *Ho* rejected? No

Which type error is possible? A Type II error is possible with prob. unknown $= \beta$.

State a conclusion in written form in the context of the experiment.

We cannot conclude at the $\alpha = 0.05$ level that the median caffeine level exceeds 300 mg per cup.
ie, we have insufficient evidence to conclude that the median caffeine level exceeds 300 mg per cup.