Example. Building specifications in a city require that the mean breaking strength of residential sewer pipe be more than 2,400 lbs/ft of length. Each manufacturer that wants to sell pipe to the city must show that its pipe meet specifications.

For each manufacturer, we want to decide whether the mean breaking strength of its pipe exceeds 2,400 lbs/ft.

Suppose we test 50 sections of a manufacturer's sewer pipe and find the mean of the 50 measurements to be $\bar{x}$ = 2,460 with a standard deviation of s = 200 lbs/ft. Do these results provide strongly convincing evidence that the population mean exceeds 2,400 lbs/ft? Strongly convincing evidence will exist when the value of $\bar{x}$ exceeds 2,400 by an amount that cannot readily attributed to sampling variability.

CI Approach:

Let's calculate a 90% CI for the population mean.

$$\bar{x} \pm z_{\frac{0.10}{2}} \frac{s}{\sqrt{n}} = 2{,}460 \pm 1.645 \frac{200}{\sqrt{50}} \approx 2{,}460 \pm 46.528$$

So, our 90% CI is (2413.5,2506.5).
What can do say in this case?

In a similar manner we can find that a 95% CI is (2404.6, 2515.4)
What do we say in this case?

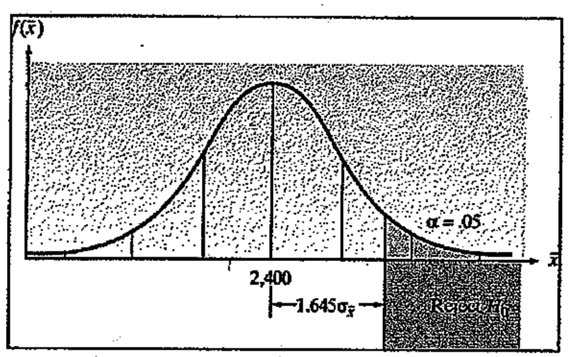
Hypothesis Testing Approach:

We define a pair of hypotheses as follows-

- The manufacturer's pipe fails to meet specifications.
  Null Hypothesis (H_0): $\mu \leq 2{,}400$
- The manufacturer's pipe meets specifications
  Alternative (Research) Hypothesis (H_a): $\mu > 2{,}400$

We compute a **test statistic** which is the z-value that measures the distance between our value of $\bar{x}$ which is 2,460 and the value of μ specified in the null hypothesis. (In cases like this one where the null hypothesis contains more than one value of μ, we use the value of μ closest to the values of μ specified in the alternative hypothesis or 2,400).

$$z = \frac{\bar{x} - 2400}{s/\sqrt{50}} \approx 2.12$$



So, the sample mean is 2.12 standard deviations above the hypothesized of 2,400. As we consider the figure to the right we see that if the hypothesis μ = 2,400 is true the chance of observing a sample mean $\bar{x}$ of than 1.645 standard deviations above is only 0.05. (In this case, p($\bar{x} \geq 2400$ $when\ \mu = 2{,}400$) is 0.017.) We determine that either H_0 is true and a rare event has occurred or H_a is true and the sample mean actually exceeds 2,400. In this case we will probably reject the null hypothesis and conclude that the alternative hypothesis is more likely to be true.