Z-Scores and the (Continuous) Normal Distribution

In Chapter 2 we considered percentile ranking as a measure of relative standing. Recall that descriptive measures of the relationship of a measurement to the rest of the data are called **measures of relative standing**. Another measure of relative standing is the *z-score*. The z-score makes use of the mean and standard deviation of a data set to specify the location of a specific measurement.

> The **sample z-score** of a measurement x is $z = \frac{x - \bar{x}}{s}$.
>
> The **population z-score** for a measurement x is $z = \frac{x - \mu}{\sigma}$

Example: Tom experienced mixed results on two tests last week. He is happy with his score of 71 on his physics exam but he is not happy with his score of 73 on his history exam. Tom explained his happiness with his exam score in physics by saying the mean score on the physics test was 60 with a standard deviation of 8, and he explained his unhappiness with his history test score by noting that the mean score on the history exam was 80 with a standard deviation of 6. Tom remarked that his relative standing in his physics class was better than his relative standing in his history class. In comparing his z-scores on the two tests he found the following.

Physics test z-score = $\frac{71 - 60}{8} = 1.375$

History test z-score = $\frac{73 - 80}{6} \approx -1.667$

Hence his score on the physics was 1.375 standard deviations above the class mean while his score on the history test was 1.667 standard deviations below the class mean.

Tom's friend Jack received a 58 on the physics test and an 80 on the history test. What were Jack's z-scores on the two tests?

Normal (Continuous) Distribution

A commonly observed continuous random variable has bell shaped (mound shaped, symmetric) probability distribution as shown below. It is known as a **normal random variable** and its probability distribution is called a **normal distribution**.
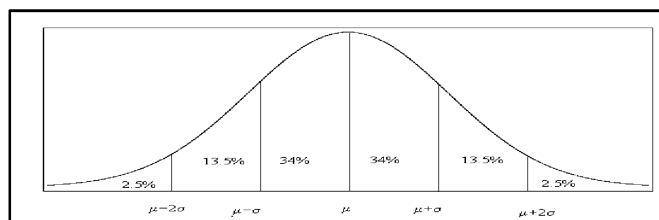

The normal distribution is symmetric about its mean μ and its spread is determined by its standard deviation σ. The notation we use when a random variable Y is normally distributed with a mean of μ and standard deviation σ is Y ~ N(μ, σ).

Here are graphs of three normal distributions.

Working left to right the three distributions
are $Y_1 = N(-4, 0.5)$, $Y_2 = N(0, 1.5)$, $Y_3 = N(3, 1)$.
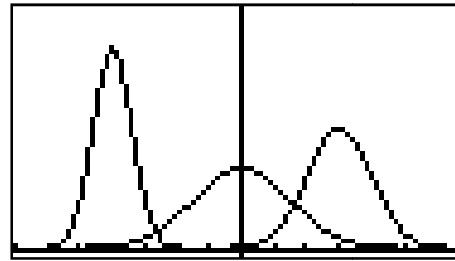
Below is a graph of a normal probability distribution showing the percent of observations we expect
to find in certain intervals about the mean.
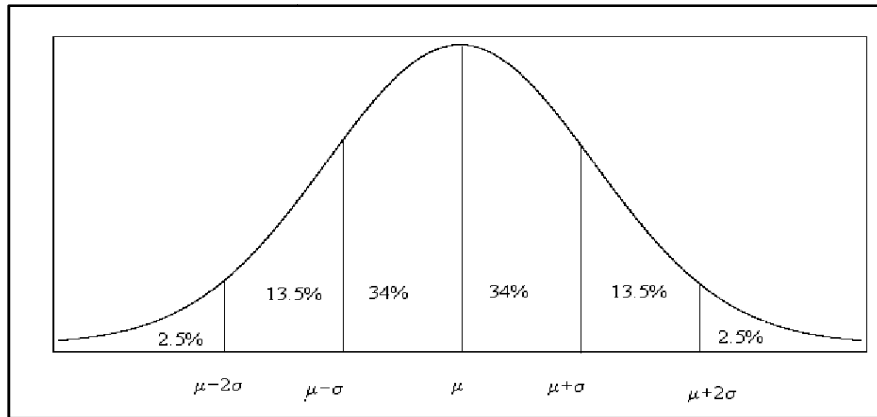
Finding Normal Probabilities

Probabilities associated with normal distributions are represented by areas under the graph of the
probability distribution. For example, in the graph above, $P(\mu - 2\sigma < x < \mu) = 0.475$ because the area
under the curve between $\mu - 2\sigma$ and μ is 0.475.

Suppose we have the normal random variable $Y_3 = N(3, 1)$ shown on the right in the graph at the top
of this page, and we seek $P(2.5 < x < 4)$. In the graph below the area of the shaded region
represents the probability we seek. The graph is produced by an applet designed by Gary H.
McClelland. (http://psych.colorado.edu/~mcclella/java/normal/accurateNormal.html)

Note, $P(2.5 < x < 4) \approx 0.5328$.

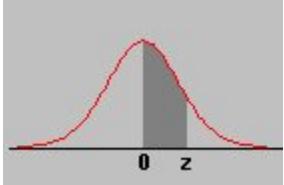We can also use MINITAB or our TI calculators to
determine normal probabilities.
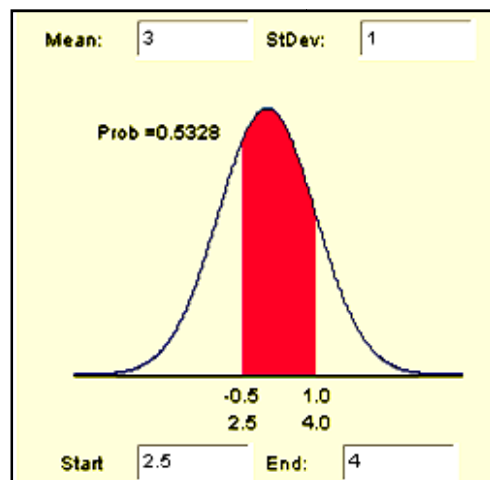
Z-Scores and the Standard Normal Distribution

The standard normal distribution is the normal distribution N(0, 1). That is, the normal distribution with μ = 0 and σ = 1. A random variable with a standard normal distribution, denoted by the symbol z, is called a standard normal random variable. In this course, we will usually convert all values of normal random variables to standard normal variables (z-scores). Table III in our text can be used to find probabilities associated with values of the standard random variable z.

For example we can determine the probability that the standard random variable z falls between 0 and 0.35.

Area between 0 and z

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879

Reading from the table we can determine that P(0 < Z < 0.35) = _____

This reading can be confirmed using the applet as above.
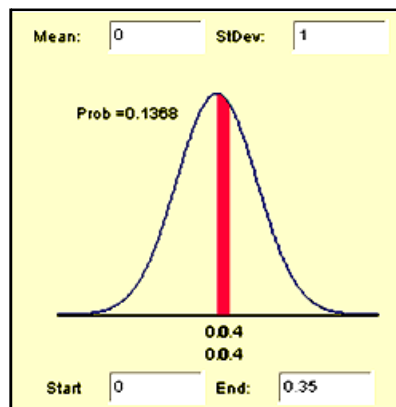
Consult pages 203-206 for instructions on using a TI graphing calculator to find standard normal probabilities and nonstandard normal probabilities.

Example: Suppose x is a normally distributed random variable with μ = 11 and σ = 2.
Find P(x > 13.24).