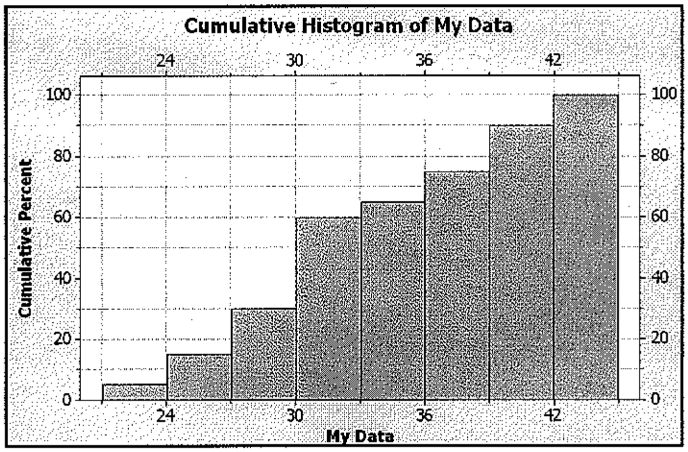
MATH 155 In-Class Work

1. Consider the cumulative histogram for a set of measurements call "My Data."



   a. About _____% of the measurements are less than 33.
   b. _____% of the measurements are at least 33.
   c. About _____% of the measurements are between 33 and 42.
   d. At most _____% of the measurements are less than 42.
   e. _____% of the measurements are at least 42.
   f. At most _____% of the measurements are less than 24.
   g. If measurements are selected at random from the data set and then replaced, about what percent of the time will the measurement selected
      - be 42 or larger? _____%
      - be between 30 and 39? _____%
      - be less than 45? _____%
      - be more than 45? _____%
      - be less than 27? _____%
      - be either less than 27 or greater than or equal to 42? _____%

2. Some Intuitive Probability

In mathematics a probability of an event is represented by a real number in the range from 0 to 1 and represents the likelihood that the event will occur. An impossible event has a probability of 0, and a certain event has a probability of 1.

Suppose the data set "My Data" of Exercise 1 above consists of the following numbers:

{23, 24, 26, 28, 28, 28, 30, 30, 32, 32, 32, 32, 34, 36, 38, 39, 40, 40, 42, 43}

Now suppose each number in the data set is written on a blank card and the deck of cards is shuffled and one card is selected at random from the deck. Assign a number in the range from 0 to 1 to each of the following events to reflect how likely the event is to occur.

a. The number 23 is selected. _____
b. The number 28 is selected. _____
c. A number less than 33 is selected. _____
d. A number 42 or larger is selected. _____
e. A number either less than 27 or greater than or equal to 42 is selected. _____
f. A number less than 44 is selected. _____
g. A number greater than or equal to 44 is selected. _____
h. An odd number less than 30 is selected. _____
i. Given that the number selected was less than 30, what is the probability is odd? _____

3. Some Probability Formal Terminology

An ***experiment*** is an act or process of observation that leads to a single outcome that cannot be predicted with certainty.

Consider the simple experiment described in Exercise 2. The most basic possible outcomes of that experiment are as follows:

Select a 23, Select a 24, Select a 26, Select a 28, Select a 30, Select a 32, Select a 34, Select a 36, Select a 38, Select a 39, Select a 40, Select a 42, Select a 43

The most basic possible outcomes of an experiment, like those above, are called ***sample points***. The ***sample space*** of an experiment is the collection of all its sample points. *The probability of a sample point is represented by a real number between 0 and 1 which measures the likelihood that the outcome will occur when the experiment is performed.*

Rules for Assigning Probabilities to Sample Points

i.) Each sample point's probability must lie between 0 and 1.
ii.) The probabilities of the sample points in a sample space must sum to 1.

An assignment of probabilities to the sample points in the experiment of this example is displayed in the following table.

Sample points	23	24	26	28	30	32	34	36	38	39	40	42	43
Probability													

An ***event*** is a collection of sample points.

For the experiment in this example, we might be interested in the probability that a number less than 33 is selected. This event can be denoted by A = {23, 24, 26, 28, 30, 32}. If we denote the probability of A, by *P*(A), then *P*(A) = _____.

The ***probability of an event*** is calculated by summing the probabilities of the sample points in A.

Find the probability of each of the following events.

a. B: A number either less than 27 or greater than or equal to 42 is selected. *P*(B) = _____
b. C: An odd number is selected. *P*(C) = _____
c. D: A number greater than 34 or an odd number is selected. *P*(D) = _____
d. E: A number greater than or equal to 44 is selected. *P*(E) = _____
e. F: A number less than 44 is selected. *P*(F) = _____
f. G: An even number greater than 38 is selected. *P*(G) = _____

4. Probability of Compound Events

The ***union*** of two events A and B is the event that occurs if A or B (or both A and B) occurs on a single performance of the experiment. We denote the union of events A and B by the symbol $A \cup B$. We denote the probability of $A \cup B$ by $P(A \cup B)$.

Consider events B and C defined above. $P(B \cup C) =$ _____

The ***intersection*** of two events A and B is the event that occurs if both A and B occur on a single performance of the experiment. We write $A \cap B$ for the intersection of A and B.

For events B and C defined above $P(B \cap C) =$ _____

The ***complement*** of an event A is the event that A does not occur. We denote the complement of A by A^c.

Describe the event that is the complement of the event C defined above.

In this case $P(C^c) =$ _____

For any event A, we can conclude that $P(A) + P(A^c) =$ _____.

Consider events C and G defined above. $P(C \cap G) =$ _____.

We call events like C and G ***mutually exclusive events***. (Why?)

Name two other mutually exclusive events from the events B, C, D,..., G defined above.

MATH 155 Assigning Probabilities to Events

Consider an experiment where pair of fair dice is rolled and the number of spots showing on the top face of each die is noted. We can suppose that one die is red and the other green. One representation of the sample space for this experiment is shown below. In each pair (g, r), the "g" is the number of spots showing on the green die and the "r" is the number of spots showing on the red die.

● 1,1	● 1,2	● 1,3	● 1,4	● 1,5	● 1,6
● 2,1	● 2,2	● 2,3	● 2,4	● 2,5	● 2,6
● 3,1	● 3,2	● 3,3	● 3,4	● 3,5	● 3,6
● 4,1	● 4,2	● 4,3	● 4,4	● 4,5	● 4,6
● 5,1	● 5,2	● 5,3	● 5,4	● 5,5	● 5,6
● 6,1	● 6,2	● 6,3	● 6,4	● 6,5	● 6,6

In the spaces adjacent to each sample point (outcome) assign a probability to each sample point.

Suppose we define events as follows:

A: A five appears on each die.
B: The sum of the numbers on the dice is seven.
C: A six appears on at least one die.
D: The sum of the numbers is at least six.
E: A six appears on the green die.

Determine the following probabilities:

a. $P(A) =$
b. $P(B) =$
c. $P(C) =$
d. $P(D) =$
e. $P(E) =$
f. $P(B \cap C) =$
g. $P(B \cup C) =$
h. $P(E^c) =$
i. ~~$P(A \cap C) =$~~
j. $P(E^c \cup E) =$
k. The probability of B given that we can see that E has occurred.
l. The probability of B given that we can see that E^c has occurred.