Probability (Continued)

Some terminology we have introduced:

Experiment
Basic outcomes
Sample points
Sample space
Probability Rules for Sample Points
Probability of an event
Compound events
Union of two events
Intersection of two events
Complementary events
Rule of complements
Mutually exclusive events

Some more terminology follows.

Recall our example of last week. We consider an experiment where pair of fair dice is rolled and the number of spots showing on the top face of each die is noted. We supposed that one die is red and the other green. One representation of the sample space for this experiment is shown below. In each pair (g, r), the "g" is the number of spots showing on the green die and the "r" is the number of spots showing on the red die.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

We associated probabilities with some events as follows:

A: A five appears on each die.	$P(A) = 1/36$
B: The sum of the numbers on the dice is seven.	$P(B) = 6/36 = 1/6$
C: A six appears on at least one die.	$P(C) = 11/36$
D: The sum of the numbers is at least six.	$P(D) = 26/36 = 13/18$
E: A six appears on the green die.	$P(E) = 6/36 = 1/6$

Events A and C are mutually exclusive because ____.

Events B and C are not mutually exclusive because _____________________________________.

As we have seen, additional knowledge may affect the probability we assign to an outcome. For example, the probability that D occurs given that we can see that E has occurred is 1. We call a probability that reflects additional knowledge, ***conditional probability***. We use the symbol $P(D|E)$ to denote the probability that D occurs given that we can see that E has occurred.

Calculate the following conditional probabilities:

P(A|D) = P(C|E) = P(B|C) = P(A|B) =

Conditional Probability Formula

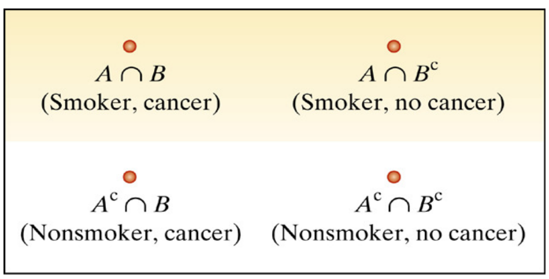
The *conditional probability that event A occurs given that event B occurs*, denoted by *P*(A|B), is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Today's Example. Consider an experiment where an individual is selected at random from the adult male population. Let A denote the event that the individual smokes. What event does A^c represent?

Also, let B denote the event that the individual develops cancer. What event does B^c represent?

The sample space *S* for this experiment is represented below in the table on the left. Probabilities for one section of the US are shown in the table on the right.



S

Probabilities of Smoking and Developing Cancer		
	Develops Cancer	
Smoker	Yes, B	No, B^c
Yes, A	0.05	0.20
No, A^c	0.03	0.72

Determine each of the following conditional probabilities and then translate each probability statement into English.

P(B|A) =

P(B^c|A) =

P(B|A^c) =

P(B^c|A^c) =

We can make use of the Conditional Probability Formula to determine the probability of the intersection of two events. Suppose we want to calculate P(AlB). Recall that

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

The ***Multiplicative Rule of Probability*** follows below.

$$P(A \cap B) = P(B)P(A|B) \text{ or equivalently } P(A \cap B) = P(A)P(B|A)$$

Show how to use the multiplicative rule of probability to calculate $P(A \cap B)$ in today's example.

In the space below we represent the situation of today's example using a device called a ***tree diagram***.

Events A and B are called ***independent events*** if the occurrence of B does not alter the probability that event A has occurred; that is A and B are independent if $P(A \mid B) = P(A)$. Events that are not independent are called ***dependent***.

Are the events A and B of our current example independent? (Justify.)

Are the events B and E of last week's example independent? (Justify.)