Z-Scores

We have considered percentile ranking as a measure of relative standing. Recall that descriptive measures of the relationship of a measurement to the rest of the data are called **measures of relative standing**. Another measure of relative standing is the *z-score*. The z-score makes use of the mean and standard deviation of a data set to specify the location of a specific measurement.

The **sample z-score** of a measurement x is $z = \frac{x - \bar{x}}{s}$.

The **population z-score** for a measurement x is $z = \frac{x - \mu}{\sigma}$

Example: Tom experienced mixed results on two tests last week. He is happy with his score of 71 on his physics exam but he is not happy with his score of 73 on his history exam. Tom explained his happiness with his exam score in physics by saying the mean score on the physics test was 60 with a standard deviation of 8, and he explained his unhappiness with his history test score by noting that the mean score on the history exam was 80 with a standard deviation of 6. Tom remarked that his relative standing in his physics class was better than his relative standing in his history class. In comparing his z-scores on the two tests he found the following.

Physics test z-score = $\frac{71 - 60}{8} = 1.375$

History test z-score = $\frac{73 - 80}{6} \approx -1.667$

Hence his score on the physics was 1.375 standard deviations above the class mean while his score on the history test was 1.667 standard deviations below the class mean.

Tom's friend Jack received a 58 on the physics test and an 80 on the history test. What were Jack's z-scores on the two tests?

Normal (Continuous) Distribution

A commonly observed continuous random variable has bell shaped (mound shaped, symmetric) probability distribution as shown below. It is known as a **normal random variable** and its probability distribution is called a **normal distribution**.

Below is a graph of a normal probability distribution showing the percent of observations we expect to find in certain intervals about the mean.