Assignment #9

5.4

"We are 95% confident that an interval estimate contains μ" means that we are 95% confident that the true population mean lies between the lower and upper bounds of the confidence interval.

5.8

a. $\bar{x} \pm 1.96(\frac{\sigma}{\sqrt{n}})$ represents a 95% confidence level

b. $\bar{x} \pm 1.645(\frac{\sigma}{\sqrt{n}})$ represents a 90% confidence level

c. $\bar{x} \pm 2.575(\frac{\sigma}{\sqrt{n}})$ represents a 99% confidence level

d. $\bar{x} \pm 1.28(\frac{\sigma}{\sqrt{n}})$ represents an 80% confidence level

e. $\bar{x} \pm 0.99(\frac{\sigma}{\sqrt{n}})$ represents a 68% confidence level

5.10 A random sample of 90 observations produced a mean of 25.9 and a standard deviation of 2.7.

a. (25.342, 26.458) is the 95% confidence interval for μ.
b. (25.432, 26.368) is the 90% confidence interval for μ.
c. (25.167, 26.633) is the 99% confidence interval for μ.

5.11 A random sample of 100 observations from a normally distributed population possesses a mean equal to 83.2 and a standard deviation equal to 6.4. I will use a TI-83+ to calculate the confidence intervals below.

a. The 95% confidence interval is (81.946, 84.454)
b. The confidence coefficient of 0.95 gives the probability that the 95% confidence interval contains the true population men.
c. The 99% confidence interval is (81.551, 84.849)
d. If the sample size is held fixed and the value of the confidence coefficient is increased, the width of the confidence interval increases.
e. Yes, the confidence intervals would still be valid even if the original population is not normal because the sampling distribution of the mean would still be normal provided the original distribution is not extremely skewed.

Applet Exercise 5.1

a. 95 of the 100 95% confidence intervals contain the mean. We would expect 95 intervals to contain the mean. 98 of the 100 99% confidence intervals contain the mean. We would expect 99 of the intervals to contain the mean.
b. We would expect the 99% confidence interval to have more intervals containing the mean because the 99% confidence interval is wider than the 95% confidence interval. That was the case in this simulation.
c. The proportion of 95% confidence intervals containing the mean approaches 0.95 and the proportion of 99% confidence intervals containing the mean approaches 0.99. This is the expected result because the meaning of the 95% confidence interval suggests that the expected proportion of confidence intervals containing the true mean is 0.95. Analogous reasoning applies to the 99% confidence interval.
d. If the original distribution is right skewed and we run the simulation 30 times, the results are similar; so we need not change our response to 5.11e.

5.16

a. Our target parameter is the mean egg length (mm) for the New Zealand bird population.
b. I used MINITAB to select 50 egg lengths from the 132 measurements in the data set.
c. The sample mean $\bar{x} = 59.46$ and standard deviation $s = 41.41$
d. A 99% confidence interval for the true mean egg length is $(44.375, 74.545)$
e. We can be 99% confident that the true mean egg length is in the interval $(44.375, 74.545)$

5.62 The sampling error SE is one-half the width of the CI.

5.70 We have sufficient funds to draw a sample of size $n = 120$

See next page for alternative approach

a) The SE for a 95% CI is $1.96\left(\frac{12}{\sqrt{120}}\right) \approx 2.15$
So our CI is 4.3 units in width.
We don't have sufficient funds to estimate the population mean with 95% confidence

b) The SE for a 90% CI is $1.645\left(\frac{12}{\sqrt{n}}\right) \approx 2 \Rightarrow n \approx 98.1$
So, we need a sample of size 99 and we have the funds.

5.80 To estimate the mean within 5 mg with 95% confidence we first have to estimate σ or s.
The range of the data in our sample is 60 to 180 or 120.
$s \approx \sigma \approx \frac{120}{4} \approx 30$. (Use $\frac{\text{range}}{4} \approx s$)
Our $n \approx \frac{(1.96)^2(2)^2}{5^2} \approx 138.3 \approx 139$. We will need 139 cups in our sample.

5.80

To estimate μ with a sampling error of 5 mg with 95% confidence we work with a sample of size n where

$$n \approx \frac{\left(Z_{\frac{0.05}{2}}\right)^2 \sigma^2}{(5)^2} \quad (\text{See p. 285})$$

We estimate σ by $\frac{\text{Range}}{4} = \frac{180-60}{4} = \frac{120}{4} = 30$ (page 285)

So, $n \approx \frac{(1.96)^2 (30)^2}{5^2} \approx 138.3$

So, we pick a sample of 139 cups to obtain an estimate of the mean caffine content within 5 mg with 95% confidence.

5.70a With \$1200 we can draw a maximum of 120 samples. We need n samples to estimate μ with 95% confidence with an interval 4 units wide when $\sigma \approx 12$. The value of n is determined by

$$n \approx \frac{\left(Z_{\frac{0.05}{2}}\right)^2 \sigma^2}{(2)^2} \approx \frac{(1.96)^2 (12)^2}{(2)^2} \approx 139$$

So, we have insufficient funds to draw a sample of size 139. Hence, we cannot make an estimate to the specified degree of precision.