Review Exercises 11/18/08

1. Suppose x is a binomial random variable with $n = 8$ and $p = 0.5$.
   a. Compute μ and σ.
   $\mu = np = 4$; $\sigma = \sqrt{np(1-p)} = \sqrt{4(.5)(.5)} = 1$
   b. Compute the probability of at least 7 successes. That is $P(x \geq 7)$.
   $P(x \geq 7) = 1 - P(x \leq 6) \approx 1 - 0.965 = 0.035$
2. Suppose x is a binomial random variable with $n = 20$ and $p = 0.5$.
   a. Compute μ and σ.
   $\mu = np = 10$; $\sigma = \sqrt{np(1-p} \approx \sqrt{20(0.25)} = \sqrt{5} \approx 2.236$
   b. Compute the probability of at least 14 successes. That is $P(x \geq 14)$.
   $P(x \geq 14) = 1 - P(x \leq 13) \approx 1 - 0.942 \approx 0.058$
3. Suppose x is a normally distributed random variable with $\mu = 4$ and $\sigma = 1$.
   a. $P(x \leq 6) \approx 0.977$
   b. $P(x \geq 7) \approx 0.00135$
4. A random sample of 100 observations from a normally distributed population possesses a mean $\bar{x}$ of 80 and a standard deviation s of 12. Specify a 90% confidence interval for μ. In this case what is our sampling error? What should our sample size be to estimate μ with a sampling error of 1.0 with 90% confidence?

90% CI: $80 \pm Z_{0.05}\left(\frac{12}{\sqrt{100}}\right) = 80 \pm 1.645(1.2) = 80 \pm 1.974 \Rightarrow (78.026, 81.974)$

SE = 1.974

Sample size we seek is $n = \frac{(1.645)^2(12)^2}{(1)^2} \approx 390$

5. A random sample of 16 observations from a population that can be assumed to be normal has a mean $\bar{x}$ of 10 and a standard deviation s of 2. Specify a 90% confidence interval for μ.

90% CI: $10 \pm t_{0.05}\left(\frac{2}{\sqrt{16}}\right) \approx 10 \pm 1.753(0.5) \approx 10 \pm 0.8765 \Rightarrow (9.1235, 10.8765)$

15 df

6. A random sample of $n = 100$ observations from a population with $s = 60$ and $\bar{x} = 110$. Test H_0: $\mu = 100$ against H_a: $\mu > 100$ using $\alpha = 0.05$. Find the p-value. Interpret your results.

One tailed Test Rejection rejection region $Z > 1.645$

$Z = \frac{\bar{x} - 100}{s/\sqrt{n}} \approx \frac{110 - 100}{60/\sqrt{100}} \approx \frac{10}{6} \approx 1.667$. Z falls in the rejection region.

p-value $\approx P(Z \geq 1.667) \approx 0.0478 < \alpha = 0.05$

Reject H_0. Evidence suggests μ is greater than 100.

7. A random sample of $n = 20$ observations from a random population with $s = 60$ and $\bar{x} = 110$. Test H_0: $\mu = 100$ against H_a: $\mu > 100$ using $\alpha = 0.05$. Find the p-value. Interpret your results.

By the t-table the one-tailed rejection region when $\alpha = 0.05$ is $t > t_{0.05}$ which is 1.729 with $n-1 = 19$ df. So, we reject H_0 if $t > 1.729$.

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \approx \frac{110 - 100}{60/\sqrt{20}} \approx 0.745$. t is not in the rejection region.

p-value = $0.233 > \alpha = 0.05$

We cannot reject H_0. We have insufficient evidence to suggest that $\mu > 100$.