4.1

54. To find the maximum revenue, I need to locate some critical values and examine different intervals on the graph to determine where the function is increasing or decreasing. First, however, I need to know the derivative of the revenue function $R(x) = -x^3 + 36x$

$R'(x) = -3x^2 + 36$ Next, to find a critical value, I set the derivative equal to zero.

$$R'(x) = 0 = -3x^2 + 36$$
$$3x^2 = 36$$
$$x^2 = 12$$
$$x = \pm 3.46$$

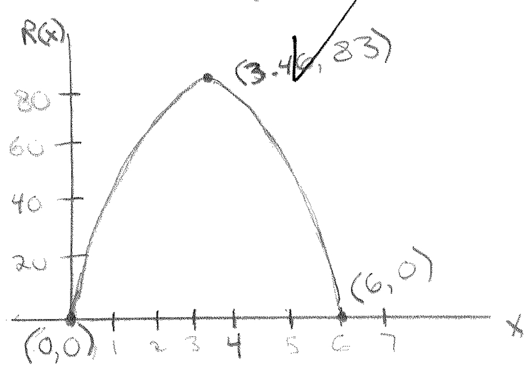
So, $x = \pm 3.46$ are critical values and I can now look at test values in three different intervals to determine where the function is increasing or decreasing and where the maximum occurs.

$[-\infty, -3.46]$ $R'(-5) = -39$ so the graph is decreasing

$[-3.46, 3.46]$ $R'(0) = 36$ so the graph is increasing

$[3.46, \infty]$ $R'(5) = -39$ so again the graph is decreasing

Therefore, the maximum revenue occurs at $x = 3.46$.
Really, there was no need to test the interval $[-\infty, -3.46]$ because there can't be negative production.



$\nearrow$ 3.46 $\searrow$

Drawing the critical value like this helps make it seem clearer.