5. (6 points) Suppose a producer has determined that its cost function, measured in dollars, is given by $C(x) = 1000 + 20x$ where x is the number of units produced and sold. The firm's revenue function, in dollars, is given by $R(x) = 200x - \frac{1}{2}x^2$.

Form B

a. Specify the profit function P(x). $P(x) = 180x - \frac{1}{2}x^2 - 1000$ ✓

b. Specify the marginal profit for any x. $P'(x) = 180 - x$ ✓

c. Find P'(100) and interpret the result.

$P'(100) = 80$ ✓ \$80 is the approximate additional profit due to the sale of the ~~100st~~ 101st unit. ✓

d. What is the producer's maximum profit? How many units produced and sold yield that maximum profit?

The maximum profit of \$15,200 occurs when 180 units are produced and sold. ✓

6. (7 points) The population of a city is increasing at the rate given by $P'(t) = 3000e^{0.04t}$, where t is time in years from the beginning of 2000 and P(t) is the population of the city t years after the beginning of 2000.

Form B

a. At what rate was the population growing at the beginning of 2003?

$P'(0) = 3000e^{0.04(3)} = 3382$ ✓

So, at the beginning of 2003 the population is growing by 3382 people/Year. ✓

b. Show how to use integration to help you determine the change in population from the beginning of 2001 to the beginning of 2004.

$$\int_1^4 3000e^{0.04t}\,dt = 75000e^{0.04t}\Big|_1^4 \approx 9953$$ ✓

So, the population increases by 9953 between the beginning of 2001 and the beginning of 2004. ✓

c. What was the city's population at the beginning of 2000?

$P(t) = 75000e^{0.04t}$ So $P(0) = 75{,}000$ ✓

So, the city's population at the beginning of 2000 was 75,000 ✓