Review Ex, Ch 4

#52

Let x = the amount charged each customer ($x \geq 60$)

Let $N(x)$ = the number of customers when each customer is charged \$$x$.

So, by the information in the exercise,

$N(x) = [40 - \frac{1}{2}(x-60)]$

Because the company's revenue is the product of the amount charged each customer and the number of customers we have

$R(x) = x\,N(x) = x\,[40 - \frac{1}{2}(x-60)]$

where $R(x)$ = the revenue when each customer is charged \$$x$.

Simplifying we get

$R(x) = 70x - \frac{1}{2}x^2, \quad x \geq 60$

The company's cost function, $C(x)$, is its fixed costs added to its variable costs. The fixed costs are \$3000 and its variable costs are \$$4N(x)$.

So $C(x) = 3000 + 4[40 - \frac{1}{2}(x-60)]$

$C(x) = 3280 - 2x$

The company's profit function $P(x)$ is defined by

$P(x) = R(x) - C(x)$.

So, $P(x) = [70x - \frac{1}{2}x^2] - [3280 - 2x]$

$P(x) = -\frac{1}{2}x^2 + 72x - 3280, \quad x \geq 60$

Let's find where $P'(x) = 0$.

$P'(x) = -x + 72$. So, $P'(x) = 0$ when $x = 72$.

Since $P'(x) > 0$ for $60 \leq x < 72$ and $P'(x) < 0$ for $x > 72$

It follows that $P(72)$ is the company's maximum profit. Hence, profit is maximized when each customer is charged \$72.