

4. (5 points) Show how to use the concept of the derivative to help you find the equation of the line tangent to the graph of $y = 10x^3$ at the point $(2, 80)$.

$$\frac{dy}{dx} = 30x^2 \quad \text{If } x=2, \frac{dy}{dx} = 120.$$

So, the equation of the tangent line is

$$y = 120x - 160.$$

5. (10 points) Specify the derivative of each of the following:

a. $y = 8x^3 - 6x^2 + 10x - 11$

$$\frac{dy}{dx} = 24x^2 - 12x + 10$$

b. $f(x) = 30e^x$

$$f'(x) = 30e^x$$

c. $g(x) = \frac{1}{x^2} = x^{-2}$

$$g'(x) = -2x^{-3} = -\frac{2}{x^3}$$

d. $h(x) = (2x + 3)^2$

$$h'(x) = 2(2x + 3)(2)$$

$$h'(x) = 4(2x + 3)$$

$$h'(x) = 8x + 12$$

e. $y = 1000e^{0.05x}$

$$\frac{dy}{dx} = 50e^{0.05x}$$

6. (5 points) Suppose a cost function is given by $C(x) = 0.1x^3 - x^2 + 100x + 200$ where x is the number of units produced and $C(x)$ is the cost in dollars to produce units.

- a. Find $C(10)$ and interpret its meaning.

$$C(10) = 100 - 100 + 1000 + 200 = 1200$$

If costs \$1200 to produce the first 10 units.

- b. Find $C'(10)$ and interpret its meaning.

$$C'(x) = 0.3x^2 - 2x + 100$$

$$C'(10) = 30 - 20 + 100 = 110$$

If costs \$110 to produce the 11th unit.