Finding Patterns and Working with Sequences

In Activity Set 1.2 you generated some number sequences and looked for patterns that would allow you to predict subsequent terms in the sequence.

Exercise 1. The first sequence you derived from tile patterns was 1, 3, 5, 7, 9, ... the sequence of odd whole numbers.

The 1st term in the sequence is 1.

The 2nd term in the sequence is 3.

The 3rd term in the sequence is 5.

The 4th term in the sequence is 7.

The 20th term in the sequence is .

(1) Each term in the sequence is _____ the preceding term, and the first term is 1.

If we let On denote the nth term in the sequence then

$$O_1 =$$
____; $O_2 =$ ____; $O_3 =$ ____; $O_4 =$ ____; ...; $O_{20} =$ ____.

We can rewrite statement (1) as follows:

(2)
$$O_n = O_{n-1} + \underline{\hspace{1cm}}$$
, and $O_1 = 1$.

We also found that we could calculate the value of the nth term in the sequence by the following rule. (Write a sentence.)

(3)_____

We can rewrite statement (3) as follows:

(4)
$$O_n =$$
_____.

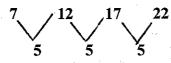
We can use statement (4) to calculate the 50th term is the sequence as follows:

$$O_{50} =$$

Rules like those in (2) above are called difference equations. Rules like those in (4) above are called functional equations. We will use both kinds of rules in problem solving.

Exercise 2. In Activity Set 1.2, we also considered the sequence

Like with the previous example, it was helpful to examine the difference between successive terms



A sequence where each successive term is obtained from the previous number by adding a fixed number is called an *arithmetic sequence*.

So, if we denote the n^{th} term in the sequence by H_n , H_n is simply ______ H_{n-1} .

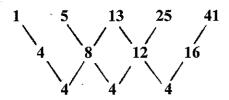
Write a difference equation for this sequence, and show how to use your difference equation to find H₇.

Write a functional equation for this sequence, and show how to use your functional equation to find H_{50} .

Exercise 3. Later in Activity Set 1.2 you generated the sequence

1 5 13 25 41

In this case, it was helpful to look at both the differences and the differences in the differences.



First Differences

Second Differences

Let's denote the n^{th} term of this sequence by Q_n . Show how to extend the pattern above to find Q_6 and Q_7 .

$$Q_6 = ___; Q_7 = ___.$$

Can you describe how Q_n is obtained from Q_{n-1} ? (Can you write a difference equation for this sequence? Later we will consider how we might find a functional equation for the sequence.)

Exercise 4. Consider the sequence 3, 6, 12, 24, 48, ... How might we describe the pattern evident in the sequence?

Suppose we denote the nth term of this sequence by G_n . It follows that $G_1 = ____, G_2 = ____, G_3 = ____,$ etc.

Can you write a difference equation explaining how G_n is obtained from G_{n-1}?

Can you write a functional equation for the sequence? If so, use your equation to calculate G_{10} .