(handwritten, circled) 100

(handwritten, top right)
16
18
16
/
50

1. (8 points) Consider the relationship expressed in the table below.

U	V
0	4
1	9
2	14
3	19
4	24
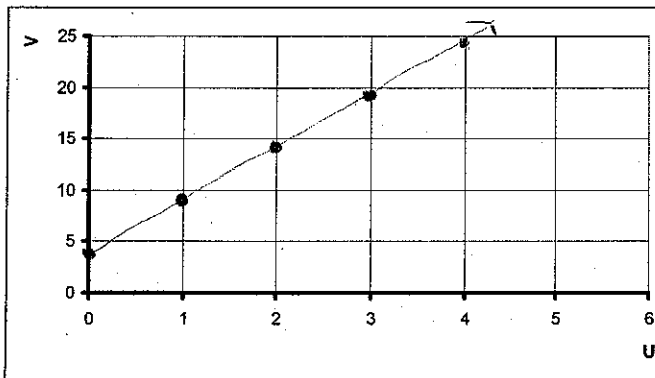
a. Plot the data points using the grid provided above.

b. Use the space below to explain and show how to determine a functional equation for the relationship between the values of V and U.

u	0	1	2	3	4
v	4	9	14	19	24

5 5 5 5

$V = 5u + 4$ ✓

The equation is $V = 5u + 4$ because the common difference is 5 for v and 1 for u and the slope is $\frac{\Delta V}{\Delta u}$, which equals $\frac{5}{1}$ or 5. Then the v-intercept is the value for v when u is zero, which is 4 in our chart

c. Show how to use your functional equation to find the value of V when U = 10. So the equation is $V = 5u + 4$.

$V = 5u + 4$
$V = 5(10) + 4$
$\boxed{V = 54}$ ✓ V equals 54 when u equals 10.

d. Show how to use your functional equation to find the value of U when V = 89.

$V = 5u + 4$
$89 = 5u + 4$
$85 = 5u$
$\boxed{u = 17}$ ✓ U is 17 when V is 89.

2. (8 points) Consider the relationship expressed in the table below. Assume the observed pattern of differences continues indefinitely.

n	0	1	2	3	4	5	6	7
C_n	4	7	14	25	40	59	82	109

3 7 11 15 19 23 27
4 4 4 4 4 4

a. Complete the table by entering the values for C_0, C_6, and C_7.

b. Complete the following rule that specifies a difference equation for the relationship.

$C_0 = 4$

$C_n = C_{n-1} + (4n - 1)$

c. Complete the following rule to specify an explicit functional equation for this relationship.

$C_n = 2n^2 + n + 4$

3. (10 points) Plot the data below on the grid provided for that purpose. Sketch a line that you think provides a good fit to the data.

    a. Explain why you drew the line where you did.
    b. Show how to find an equation for **the line you drew**. Write out that equation.
    c. Show how to use your equation to estimate a predicted value for y when x = 15.
    d. Use a numerical criterion in commenting on how well you think your line fits the data (average error.)

10

x	y
1	4
3	10
4	11
5	11
7	16
9	22
15	

a) I drew the line where I did because I was trying to get my line close to as many data points as possible.
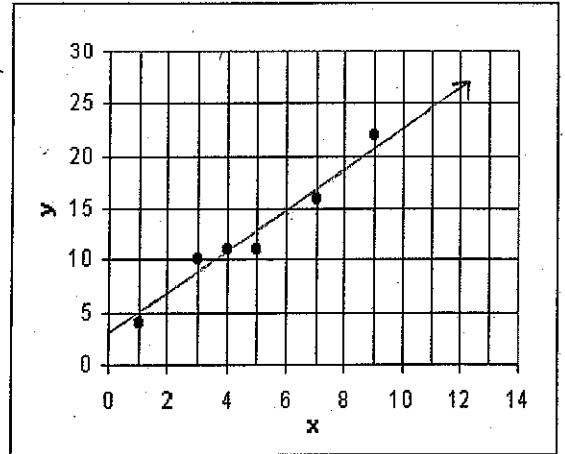
b) $(1,5)$ $(6,15)$

$$m = \frac{15-5}{6-1} = \frac{10}{5} = 2$$

$$y = mx + b$$
$$5 = 2(1) + b$$
$$5 = 2 + b$$
$$b = 3$$

$$\boxed{y = 2x + 3}$$

My equation is $y = 2x+3$.

c) $y = 2x + 3$
$y = 2(15) + 3$
$y = 30 + 3$

$$\boxed{y = 33}$$

When x is 15, y is 33.

d)

X	Y	2x+3	Error
1	4	5	1
3	10	9	1
4	11	11	0
5	11	13	2
7	16	17	1
9	22	21	1

Avg error: 1

I found my average error to be one so my line does an okay job at fitting the data.

4. (8 points) Suppose $1000 is placed in an account that pays interest at the rate of 8% per annum compounded at the end of each year. Once the money is deposited, it is left to grow with no further deposits or withdrawals.

Let A_0 = the value of the initial deposit, and
let A_n = the value of the account at the end of n years.

8

    a. Show how to calculate A_1 and A_2.

$A_1 = 1000 + 1000(.08)$ $A_2 = 1080 + 1080(.08)$
$A_1 = \$1080$ $A_2 = \$1166.40$

    b. Write a difference equation for the relationship between A_n and n.

$A_0 = 1000$
$A_n = 1.08(A_{n-1})$

    c. Write an explicit functional equation for the relationship between A_n and n.

$$A_n = 1000 \times (1.08)^n$$

    d. Show how to determine the number of years it will take for the value of the account to reach or exceed $1500.

$$A_n = 1000 \times (1.08)^n$$

$$1500 = 1000 \times (1.08)^n$$

$$1.5 = 1.08^n$$

~~$\log$~~ omit

$$n = 5.27 \approx 6$$

It will take 6 years for the value of the account to reach or exceed $1500.

5. (8 points) A racquetball club charges $30 per month plus $5 per hour for each hour of court time, and only whole numbers of hours can be purchased. Suppose x represents the number of hours of playing racquetball in a given month, and c(x) represents the total cost to the member for playing racquetball in a given month. (a) Write a functional equation for determining the cost per month of playing racquetball at the club. (b) Show how to use your functional equation to find the cost of playing 15 hours of racquetball in a given month. Clearly state your conclusion using a complete sentence. (c) If Tom is a member of the club and was charged $75 for playing racquetball during the month of October, show how to use your functional equation to determine how many hours of court time Tom used during October. Clearly state your conclusion using a complete sentence.

a) A functional equation for this is $c(x) = 5x + 30$.

b) $c(x) = 5x + 30$
$c(x) = 5(15) + 30$
$c(x) = 105$

It will cost $105 to play 15 hours of racquetball in a given month.

c) $c(x) = 5x + 30$
$75 = 5x + 30$
$45 = 5x$
$x = 9$

If Tom was charged $75 for playing racquetball during the month of October, then he played 9 hours of racquetball.

6. (8 points) We have already investigated the relationship between the Fahrenheit and Celsius temperature scales. We have seen that when the Fahrenheit temperature increases by 9 degrees the Celsius temperature increases by 5 degrees. Of course, we know that water freezes at 0^0 Celsius and at 32^0 Fahrenheit.

a. Let C = degrees Celsius and F = degrees Fahrenheit. Complete the functional equation we can use to convert from degrees Celsius to degrees Fahrenheit. Show your work in deriving the equation.

$F = \frac{9}{5}C + 32$

$m = \frac{\Delta F}{\Delta C} = \frac{9}{5}$ $F = \frac{9}{5}C + b$
$32 = \frac{9}{5}(0) + b$
$b = 32$

b. Use your equation from part (a.) to determine the following:

50^0 Celsius = 122^0 Fahrenheit

$F = \frac{9}{5}C + 32$
$F = \frac{9}{5}(50) + 32$
$F = 122^0$

-40^0 Fahrenheit = -40^0 Celsius

$F = \frac{9}{5}C + 32$
$-40 = \frac{9}{5}C + 32$
$-72 = \frac{9}{5}C$
$C = -40$