Other Exercises

Graphs on the third, fourth, and fifth attached pages.

a)

n	0	1	2	3	4	5	6	7	8
f(n)	5	10	19	32	49	70	95	124	157

1st diff.   5   9   13   17   21   25   29   33

2nd diff.   4   4   4   4   4   4   4

Sequence a is neither an arithmetic sequence nor a geometric sequence because it does not have a common difference or a common ratio. Using the method of finite differences shown above, I found f(8) to be equal to 157.

A difference equation is $\begin{cases} f(0) = 5 \\ f(n) = f(n-1) + (4n+1) \end{cases}$

You know it starts at 5 and then I figured out you add 4n+1 to the previous one.

An explicit functional equation is $f(n) = 2n^2 + 3n + 5$.

I knew since it had a second difference it followed the quadratic formula. I knew the last part was plus 5 since f(0) = 5 and I guessed and checked to get the rest.

b)

n	0	1	2	3	4	5	6	7	8
f(n)	3	6	12	24	48	96	192	384	768
ratio:		2	2	2	2	2	2	2	2

Sequence b is a geometric sequence because it has a common ratio of 2. F(8) is 768. I found this by multiplying the previous answer by 2 since the common difference is 2. A difference equation for this sequence is $\begin{cases} f(0) = 3 \\ f(n) = 2(f(n-1)) \end{cases}$
Since the sequence starts at f(0)=3 and to get the next term, you multiply the previous term by 2. An explicit functional equation for this sequence is $f(n) = 3 \times 2^n$. Since it is a geometric sequence it follows an exponential form. The 3 is in the equation because that is where it starts and the 2 is raised to the n because that is the common difference.

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c)

n	0	1	2	3	4	5	6	7	8
f(n)	4.00	5.50	7.00	8.50	10.00	11.50	13.00	14.50	16.00
Difference		1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50

Sequence c is an arithmetic sequence since it has a common difference, which is 1.50. $F(8)$ is 16.00, I found this by taking the previous term and adding the common difference (1.50) to it. A difference equation for this sequence is $\begin{cases} f(0) = 4.00 \\ f(n) = f(n-1) + 1.50 \end{cases}$ since it starts at $f(0) = 4.00$ and to find the next term you add 1.50 to the previous term. An explicit functional equation is $f(n) = 1.5n + 4.00$ since it starts at 4.0 and goes up by 1.5 each time.