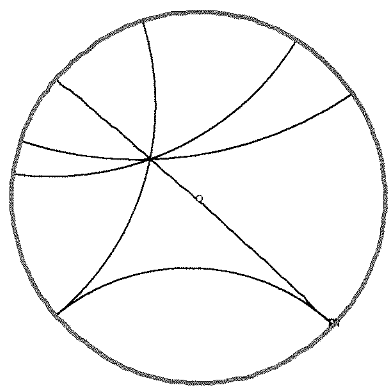
Some Notes on Hyperbolic Geometry

The axioms of neutral geometry imply that for every line *l* and for every point P not on *l* there exists at least one line through P that is parallel to *l*. If we build a system that includes the axioms of neutral geometry together with the following hyperbolic parallel postulate, that system is the basis for *hyperbolic geometry*.

Hyperbolic Parallel Postulate: Through a given point not on a given line, more than one parallel may be drawn to the given line.

Poincare's Disk Model for Hyperbolic Geometry



Given a circle Σ in the Euclidean plane, H-*points* are Euclidean points in the interior of Σ. Those H-points comprise the H-*plane*. Euclidean points on Σ are called *omega* (Ω) points of the H-plane. H-*lines* are diameters of Σ without their endpoints and arcs of circles inside Σ that meet Σ orthogonally.

Distance in Hyperbolic Geometry

For each pair of H-points C and D, CD denotes the usual Euclidean distance from C to D. The H-*distance* from C to D, denoted by d(C, D), is defined by

$$D(C , D) = \left| \ln((CA/CB)/(DA/DB)) \right|.$$