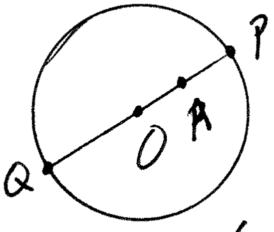
3.2
II 8

Prove: The hyperbolic distance from any point in the Interior of circle Σ to the circle itself is infinite.

Proof Suppose point A is in the interior of unit circle Σ whose center is at O.



The hyperbolic distance we seek is $d(A,P)$.
Suppose A's Euclidean distance from O is $r<1$.

We know $d(O,P) = d(O,A) + d(A,P)$.

$$d(O,P) = \lim_{s \to 1} \left| \ln \frac{1+s}{1-s} \right| = \infty$$

But $d(O,A) = \left| \ln \frac{1+r}{1-r} \right| = k$ for some finite $k \in \mathbb{R}$ because $k<1$.

So, $d(A,P) = d(O,P) - d(O,A)$
$= \infty - k = \infty$ for any finite k.

Hence, $d(A,P) = \infty$.

Consequently, the hyperbolic distance from any point in the interior of Σ to the circle itself is infinite.