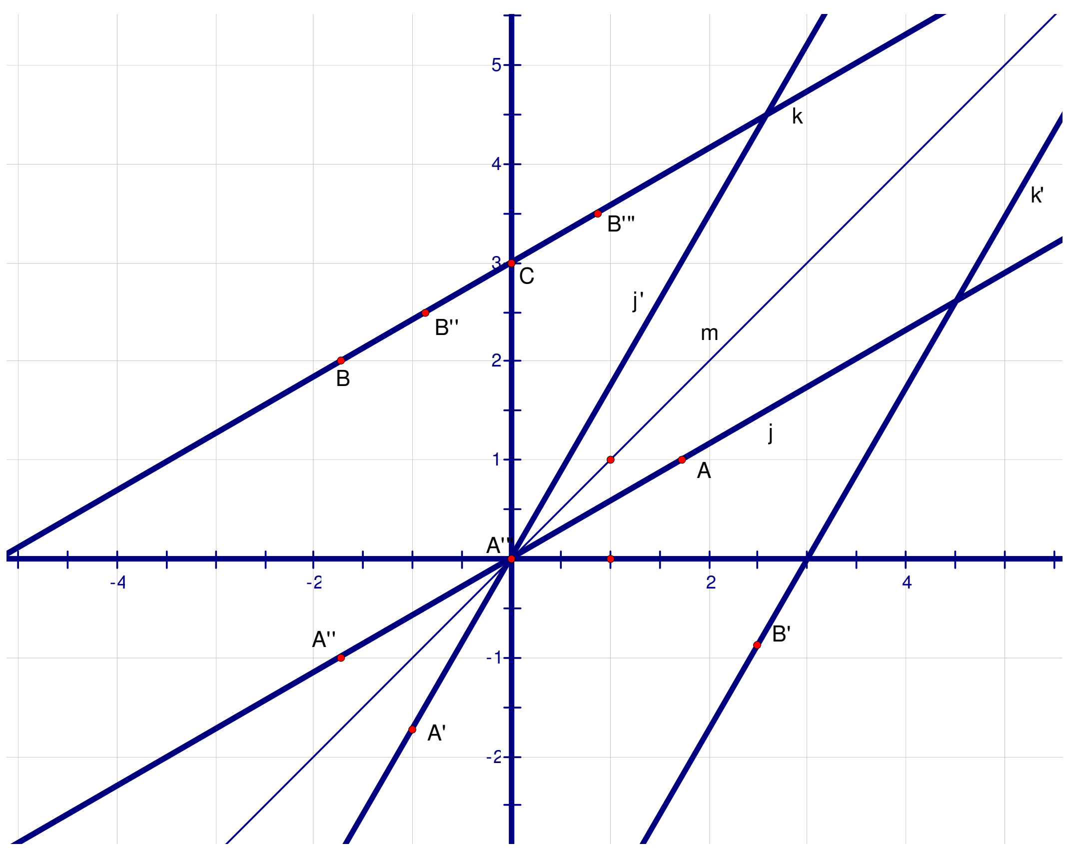
4.5 Exercises #10

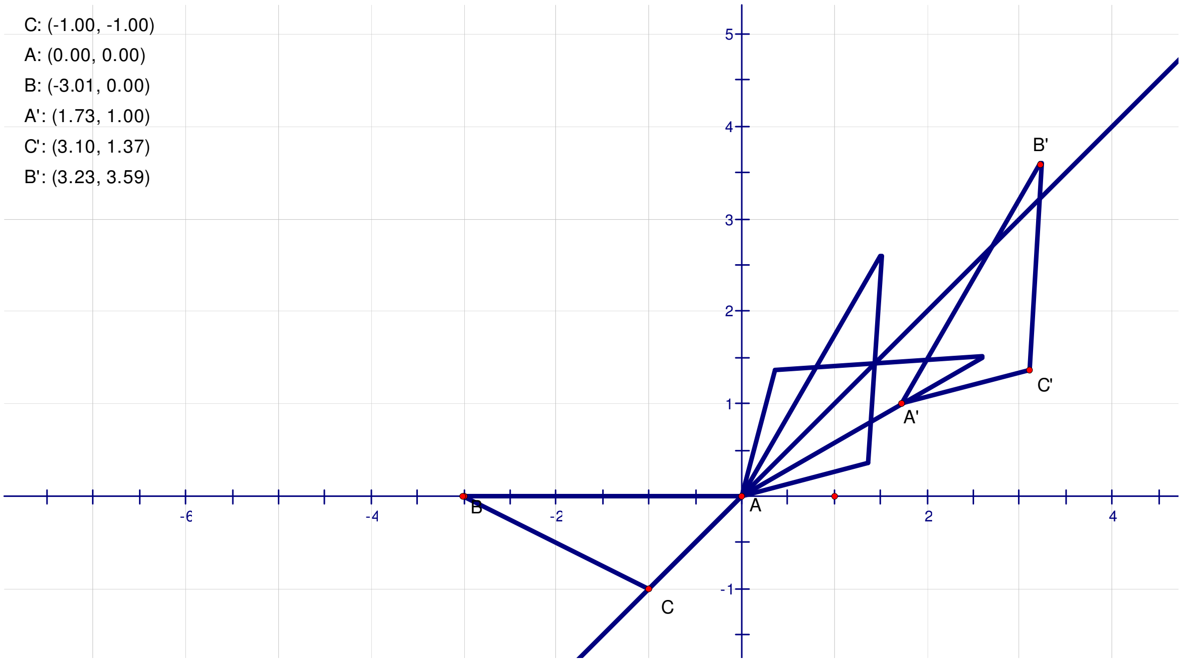
a. $T = \begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} & \sqrt{3} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b. We seek $\overline{u}$ such that $T(\overline{u}) = \begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} & \sqrt{3} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \overline{u}$. All $\overline{u} = \begin{bmatrix} \frac{2-t}{\sqrt{3}} \\ t \\ 1 \end{bmatrix}$ for $t \in R$ are invariant.

c. We seek $\begin{bmatrix} a & b & c \end{bmatrix}$ such that $\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} & \sqrt{3} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = k\begin{bmatrix} a & b & c \end{bmatrix}$

The lines $\begin{bmatrix} 1 & -\sqrt{3} & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$ are invariant under T. By part (b), the line $\begin{bmatrix} 1 & \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{3}} \end{bmatrix}$ is also invariant.





b. T is composed of the following transformations:

Rotate 210^{o} about the origin

Reflect in the line y = x

Translate through ($\sqrt{3}$, 1)

$$T = \begin{bmatrix} 0 & 0 & \sqrt{3} \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$