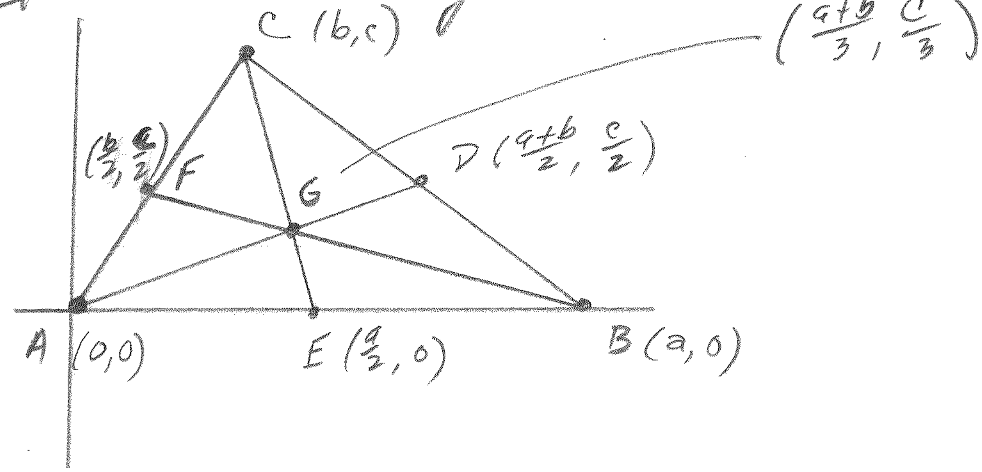
Conjecture: The three medians of a triangle are concurrent at a point that lies $2/3$ of the length from each vertex to the midpoint of the opposite. That is, the point of concurrency partitions each median in a $2:1$ ratio.

Proof Consider the figure below.



We have previously established the concurrency of the medians shown at the point $G\left(\frac{a+b}{3}, \frac{c}{3}\right)$. So, we simply apply the distance formula and calculate the ratios: AG/GD, BG/GF, and CG/GE.

$$\frac{AG}{GD} = \frac{\left[\left(\frac{a+b}{3}\right)^2 + \left(\frac{c}{3}\right)^2\right]^{1/2}}{\left[\left(\frac{a+b}{6}\right)^2 + \left(\frac{c}{6}\right)^2\right]^{1/2}} = \frac{\frac{1}{3}\left[(a+b)^2 + c^2\right]^{1/2}}{\frac{1}{6}\left[(a+b)^2 + c^2\right]^{1/2}} = \frac{6}{3} = \frac{2}{1} \checkmark$$

$$\frac{BG}{GF} = \frac{\left[\left(\frac{2a-b}{3}\right)^2 + \left(\frac{c}{3}\right)^2\right]^{1/2}}{\left[\left(\frac{2a-b}{6}\right)^2 + \left(\frac{c}{6}\right)^2\right]^{1/2}} = \frac{\frac{1}{3}\left[(2a-b)^2 + c^2\right]^{1/2}}{\frac{1}{6}\left[(2a-b)^2 + c^2\right]^{1/2}} = \frac{6}{3} = \frac{2}{1} \checkmark$$

$$\frac{CG}{GE} = \frac{\left[\left(\frac{2b-a}{3}\right)^2 + \left(\frac{2c}{3}\right)^2\right]^{1/2}}{\left[\left(\frac{2b-a}{6}\right)^2 + \left(\frac{2c}{6}\right)^2\right]^{1/2}} = \frac{\frac{1}{3}\left[(2b-a)^2 + (2c)^2\right]^{1/2}}{\frac{1}{6}\left[(2b-a)^2 + (2c)^2\right]^{1/2}} = \frac{6}{3} = \frac{2}{1} \checkmark$$

Hence, the calculations above verify our conjecture!