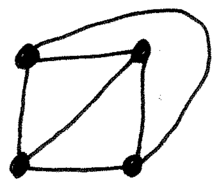
2. (15 points) Consider the following axiom system.

Undefined Terms: point, line, on
Axiom 1: Any two distinct points are on exactly one line.
Axiom 2: There exist at least three points not all of which are on the same line.
Axiom 3: Each line is on at least two distinct points.
Axiom 4: If ℓ is a line and P is a point not on the line, there exists another line that is on P but is not on any point that is also on ℓ.

(a) Verify the consistency of this axiom system with the use of a model.

dots ⟷ points
curves ⟷ lines



The existence of a model establishes the consistency of our axiom system.

(b) Show that Axiom 2 is independent of the other axioms. Start by explaining your strategy.

We establish the independence of A2 by displaying a model satisfying A1, A3, A4 but not A2.

Model 1: the empty set

model 2: 

(c) In this geometry prove or disprove: There exists at least four points

By A2 there exists 3 points, say A, B, C, that are not on the same line. By A1, A and B are on the same line ℓ. By A4 there is another line m that contains C that has no points in common with ℓ. By A3, line m must contain a point D other than C. Of course D is neither A nor B, so D is a fourth point. Consequently there exists at least four points.