3. (5 points) Define the term "neutral geometry" and specify two distinctly different models for neutral geometry. Explain how you know your two models are distinctly different.

A plane geometry is neutral if it does not include a parallel postulate or an equivalent postulate.

✓ Euclidean Geometry

✓ Hyperbolic Geometry

are two models for neutral geometry.

In Euclidean geometry we have the existence of unique parallels. In Hyperbolic geometry we do not have unique parallels — we have many parallels to a given line that pass through any point not on the given line.

4. (5 points) Prove that the following statement is a theorem in any incidence geometry that has I-1, I-2, ..., I-7 as axioms. Use exemplary proof style in your argument.

If two lines intersect, then there is exactly one plane that contains them both.

Suppose ℓ_1 and ℓ_2 are two lines that intersect. ℓ_1 and ℓ_2 have a point R in common. By I-2 ℓ_1 and ℓ_2 each have at least one other point. Suppose A is a point other than R on ℓ_1 and B is a point other than R on ℓ_2. R, A, and B are not collinear. If they were collinear, then ℓ_1 and ℓ_2 would be the same line by I-1. Axiom I-4 implies there is a unique plane Π containing R, A, and B. By I-5, ℓ_1 and ℓ_2 are both in that plane. That plane must be unique because it contains R, A, and B. So, if two lines intersect, there is exactly one plane that contains them both.