

5. (5 points) Given an axiom system with our axioms of connection (incidence) and betweenness (order), prove that there is an infinite number of points on each line. Use exemplary proof style in your argument.

Suppose there exists a line with just a finite number k of points and no more. Those points can be ordered and labeled so that $P_1 - P_2 - \dots - P_{k-1} - P_k$ are the only points. However O1 guarantees that there exists a $(k+1)$ st point P_k between P_1 and P_2 .
~~✗~~ We can't have exactly k and $k+1$ points on our line.

Consequently, there cannot be a line with just a finite number of points.
That is, each line has infinitely many points.

6. (5 points) Define each of the following terms using only the concepts of synthetic geometry:

a. segment A segment consists of two points of a line together with all the points between them.

b. ray - A ray consists of two points A and B together with all the points between A and B and all points C such that $A-B-C$.

c. interior of an angle The interior of $\angle ABC$ is the convex set formed by the intersection of the half-plane on the C side of \overleftrightarrow{AB} and the half-plane on the A side of \overleftrightarrow{BC} .

d. perpendicular If O is a point on a line l and OR is a ray or segment with its endpoint at O and if OR and l form two congruent angles, then l and OR are said to be perpendicular.