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Prove: The transformation matrix for a direct dilation with center $(h,k,1)$ and ratio r is
$$D=\begin{pmatrix} r & 0 & h(1-r) \\ 0 & r & k(1-r) \\ 0 & 0 & 1 \end{pmatrix}.$$

Proof Suppose $T:E^2\to E^2$ is a direct dilation with center $(h,k,1)$ and ratio r. T can be expressed as the composition of the following three transformations in the given order:

translate through $(-h,-k)$

dilate with center $O(0,0,1)$ and ratio r

translate through (h,k)

So, the transformation matrix D for T can be expressed as
$$D=\begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$
$$=\begin{pmatrix} r & 0 & h \\ 0 & r & k \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}=\begin{pmatrix} r & 0 & -rh+h \\ 0 & r & -rk+k \\ 0 & 0 & 1 \end{pmatrix}$$

So
$$D=\begin{pmatrix} r & 0 & h(1-r) \\ 0 & r & k(1-r) \\ 0 & 0 & 1 \end{pmatrix}$$

Consequently, the transformation matrix for a direct dilation with center $(h,k,1)$ and ratio r is
$$D=\begin{pmatrix} r & 0 & h(1-r) \\ 0 & r & k(1-r) \\ 0 & 0 & 1 \end{pmatrix}.$$