(3) Consider the strain S with axis $[1 \ 2 \ 0]$ and ratio 3.

(3a) We find a matrix for S.

The axis for the strain has equation $y = \frac{1}{2}x$

So, the angle θ that the axis of the strain makes with the x-axis $[0 \ 1 \ 0]$ is $\arctan\left(\frac{1}{2}\right)$. That is,

$\sqrt{5}$ 1 θ 2

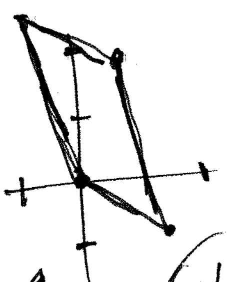
S can be expressed as the composition of 3 transformations in the order

- rotate through $-\arctan\frac{1}{2}$ about $O(0,0,1)$
- strain with axis $[0 \ 1 \ 0]$ and ratio 3
- rotate through $\arctan\frac{1}{2}$ about $(0,0,1)$

So, a matrix for S can be expressed by

$$S = \begin{bmatrix} \frac{2}{\sqrt5} & -\frac{1}{\sqrt5} & 0 \\ \frac{1}{\sqrt5} & \frac{2}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} & 0 \\ -\frac{1}{\sqrt5} & \frac{2}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} \frac{2}{\sqrt5} & -\frac{3}{\sqrt5} & 0 \\ \frac{1}{\sqrt5} & \frac{6}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} & 0 \\ -\frac{1}{\sqrt5} & \frac{2}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{7}{5} & -\frac{4}{5} & 0 \\ -\frac{4}{5} & \frac{13}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(3b)

Image of unit square

$$S \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{7}{5} & -\frac{4}{5} & 0 \\ -\frac{4}{5} & \frac{13}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{7}{5} & \frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{9}{5} & \frac{13}{5} \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Image of unit square