(4) Find invariant points

$$\begin{bmatrix} -\frac{5}{13} & \frac{27}{13} & 0 \\ -\frac{12}{13} & \frac{31}{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{cases} -\frac{5}{13}x + \frac{27}{13}y = x \\ -\frac{12}{13}x + \frac{31}{13}y = y \end{cases} \Rightarrow \begin{cases} -\frac{18}{13}x + \frac{27}{13}y = 0 \\ -\frac{12}{13}x + \frac{18}{13}y = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -18x + 27y = 0 \\ -12x + 18y = 0 \end{cases} \Rightarrow \begin{cases} -2x + 3y = 0 \\ -2x + 3y = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -2x + 3y = 0 \\ 0 = 0 \end{cases} \Rightarrow y = \frac{2}{3}x$$

$$\begin{bmatrix} -2 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

So all points on the line are invariant under T.

Find invariant lines

✓ Of course the line $\begin{bmatrix} -2 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$ is invariant under T. are other lines also invariant under T?

First we find T^{-1}.

$$\left(\begin{array}{ccc|ccc} -\frac{5}{13} & \frac{27}{13} & 0 & 1 & 0 & 0 \\ -\frac{12}{13} & \frac{31}{13} & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} -5 & 27 & 0 & 13 & 0 & 0 \\ -12 & 31 & 0 & 0 & 13 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$