#4b

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{27}{5} & 0 & -\frac{13}{5} & 0 & 0 \\ -1 & \frac{31}{12} & 0 & 0 & \frac{13}{12} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left[\begin{array}{ccc|ccc} 1 & -\frac{27}{5} & 0 & -\frac{13}{5} & 0 & 0 \\ 0 & -\frac{169}{60} & 0 & -\frac{13}{5} & \frac{13}{12} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{27}{5} & 0 & -\frac{13}{5} & 0 & 0 \\ 0 & -169 & 0 & -156 & 65 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -\frac{27}{5} & 0 & -\frac{13}{5} & 0 & 0 \\ 0 & 1 & 0 & \frac{12}{13} & -\frac{5}{13} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{31}{13} & -\frac{27}{13} & 0 \\ 0 & 1 & 0 & \frac{12}{13} & -\frac{5}{13} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \Rightarrow T^{-1} = \begin{pmatrix} \frac{31}{13} & -\frac{27}{13} & 0 \\ \frac{12}{13} & -\frac{5}{13} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So we seek $[A \quad B \quad C]$ such that

$$[A \quad B \quad C]\begin{bmatrix} \frac{31}{13} & -\frac{27}{13} & 0 \\ \frac{12}{13} & -\frac{5}{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} = [A \quad B \quad C]$$

$$\begin{cases} \frac{31}{13}A + \frac{12}{13}B = A \\ -\frac{27}{13}A - \frac{5}{13}B = B \end{cases} \Rightarrow \begin{cases} \frac{18}{13}A + \frac{12}{13}B = 0 \\ -\frac{27}{13}A - \frac{18}{13}B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 3A + 2B = 0 \\ 3A + 2B = 0 \end{cases} \Rightarrow A = -\frac{2}{3}B, \; C \text{ is FREE}$$

So, lines of the form $[-2 \quad 3 \quad C]\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$ are invariant. That is all lines with slope $\frac{2}{3}$ are invariant.