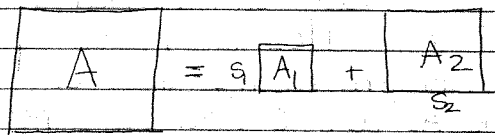
2. "An area A consisting of the sum of two squares is 1000. The side of one square is 10 less than 2/3 the side of the other square. What are the sides of the square?

To solve this problem, using algebra, first I need to read the problem carefully and draw a diagram to help picture the problem:



Next, I noted the unknown variables, s_1 for the length of the side of square A_1, and s_2 for the length of a side of square A_2. After, that, I determined what I knew about the sides of the two squares, I formed one equation: $s_1 = (2/3)s_2 - 10$, and another equation $A_1 + A_2 = 1000$. From these two equations and the knowledge that $A_1 = s_1^2$ and $A_2 = s_2^2$ I was able to substitute $s_1 = (2/3)s_2 - 10$ for s_1 in the equation: $s_1^2 + s_2^2 = 1000$ to obtain:

$$[(2/3)s_2 - 10]^2 + s_2^2 = 1000$$
$$\Rightarrow \frac{4}{9}s_2^2 - \frac{40}{3}s_2 + 100 + s_2^2 = 1000$$
$$\Rightarrow 13s_2^2 - 120s_2 + 900 = 9000$$
$$\Rightarrow 13s_2^2 - 120s_2 + (-8100) = 0 \quad \text{by algebra.}$$

Then, I used the quadratic formula to calculate s_2:

$$s_2 = \frac{120 \pm \sqrt{(-120)^2 - 4(13)(-8100)}}{2(13)}$$
$$= \frac{120 \pm \sqrt{435600}}{26}$$
$$= \frac{120 \pm 660}{26}$$

Since it is not possible for a length to be negative, I chose to use

$$\frac{120 + 660}{26} = 30.$$

Thus $s_2 = 30$ and substituting that in for $s_1 = 2/3(30) - 10$ I found $s_1 = 10$. Therefore the lengths of the sides of square A_1 and A_2 are 10 and 30, respectively.

When trying to solve this problem before the invention of algebra, I would imagine that people would use strategies such as guess and test or create a table to find possible values. I used excel to help me calculate some possible values.

x	(2/3)x-10	(x)^2	(2/3x-10)^2	(x^2)+(2/3x-10)^2
5	-6.666666667	25	44.44444444	69.44444444
10	-3.333333333	100	11.11111111	111.1111111
15	0	225	0	225
20	3.333333333	400	11.11111111	411.1111111
25	6.666666667	625	44.44444444	669.4444444
30	10	900	100	1000 ✓
35	13.33333333	1225	177.7777778	1402.777778
40	16.66666667	1600	277.7777778	1877.777778
45	20	2025	400	2425