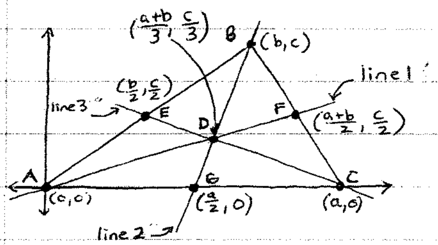
Problem: Write up your validation or rejection of our conjecture regarding the way the point in common to the medians of a triangle partitions the medians.

State conjecture at the top.

Answer:



- My validation or rejection of our conjecture will be based on the existence (or non-existence) of ~~a~~ common ratio between the lengths of the shortest and longest segments of each line.

First, we will find the lengths of the longest segments of each line. To do this, we will use the distance formula $\left(d=\sqrt{(x^2-x')^2+(y^2-y')^2}\right)$

$$|\overline{DA}| = \sqrt{\left(\frac{a+b}{3}\right)^2+\left(\frac{c}{3}\right)^2} = \sqrt{\frac{a^2+2ab+b^2+c^2}{9}} = \frac{\sqrt{a^2+2ab+b^2+c^2}}{3}$$

$$|\overline{BD}| = \sqrt{\left(\frac{2b-a}{3}\right)^2+\left(\frac{2c}{3}\right)^2} = \sqrt{\frac{4b^2-4ab+a^2+4c^2}{9}} = \frac{\sqrt{4b^2-4ab+a^2+4c^2}}{3}$$

$$|\overline{DC}| = \sqrt{\left(\frac{-2a+b}{3}\right)^2+\left(\frac{c}{3}\right)^2} = \sqrt{\frac{4a^2-4ab+b^2+c^2}{9}} = \frac{\sqrt{4a^2-4ab+b^2+c^2}}{3}$$

Second, we will find the lengths of the shortest segments of each line.

$$|\overline{FD}| = \sqrt{\left(\frac{a+b}{6}\right)^2+\left(\frac{c}{6}\right)^2} = \sqrt{\frac{a^2+2ab+b^2+c^2}{36}} = \frac{\sqrt{a^2+2ab+b^2+c^2}}{6}$$

$$|\overline{DG}| = \sqrt{\left(\frac{2b-a}{6}\right)^2+\left(\frac{2c}{6}\right)^2} = \sqrt{\frac{4b^2-4ab+a^2+4c^2}{36}} = \frac{\sqrt{4b^2-4ab+a^2+4c^2}}{6}$$

$$|\overline{ED}| = \sqrt{\left(\frac{-2a+b}{6}\right)^2+\left(\frac{c}{6}\right)^2} = \sqrt{\frac{4a^2-4ab+b^2+c^2}{36}} = \frac{\sqrt{4a^2-4ab+b^2+c^2}}{6}$$

Validation of our conjecture requires that the ratio between $|\overline{DA}|$ and $|FD|$ be the same as the ratio between $|\overline{BD}|$ and $|\overline{DG}|$ and between $|\overline{DC}|$ and $|\overline{ED}|$.

Ratio between $|\overline{DA}|$ and $|\overline{FD}|$: $\frac{\sqrt{a^2+2ab+b^2+c^2}}{3} = \frac{\sqrt{a^2+2ab+b^2+c^2}}{6}(R)$; $R = \frac{6}{3}$

Ratio between $|\overline{BD}|$ and $|\overline{DG}|$: $\frac{\sqrt{4b^2-4ab+a^2+4c^2}}{3} = \frac{\sqrt{4b^2-4ab+a^2+4c^2}}{6}(R)$; $R = \frac{6}{3}$

Ratio between $|\overline{DC}|$ and $|\overline{ED}|$: $\frac{\sqrt{4a^2-4ab+b^2+c^2}}{3} = \frac{\sqrt{4a^2-4ab+b^2+c^2}}{6}(R)$; $R = \frac{6}{3}$

We notice that all three ratios are the same. Therefore, we conclude that our conjecture regarding the way the point in common to the medians of a triangle partitions the medians is a valid conjecture.