

Write up a proof of the following: In Axiom System 2 there are at least four members and at least two committees.

So to begin this problem I knew that I must start with an existence axiom. This is axiom #3 that there is at least one committee. So here is my proof:

Theorem: There exists at least four members and at least two committees.

Proof: By Axiom 3 there exists a committee  $C_1$ .  
By Axiom 4  $C_1$  contains exactly 3 members.  
Let's call them  $a, b, c$ .

By Axiom 5 there must be a member  $d$  that is not on  $C_1$ . In other words  $d$  is distinct from the 3 members of  $C_1$ . Hence there must be at least 4 members.

By Axiom 1 there is a committee say  $C_2$  containing  $A \cup D$ .  $C_2$  cannot be  $C_1$  because  $D$  is not in  $C_1$ .

Hence we must have at least two committees. Consequently we have proved that there exists at least 4 members and two committees.

Since for this problem all we had to do was prove that there was at least four members and two committees, I started with the existence axiom and then used the others to build up to my conclusion. So after using axiom 3 I went to axiom 4, which tells us how many members each committee has. So now we have 3 members but