MATH 406   Name_____

Test #3 – Spring 2005

<u>Directions.</u>   Work on this test without the aid of any other person, and provide no aid to any other person. You may refer to your notes, textbook, or any other books you care to consult. Attach this cover sheet to your solutions and submit it with your completed test on later than 11:00 am on Monday, May 9. Be sure to cite and credit any references you use.

Work these problems in the spirit of the approach to problem solving used in this course. Pay particular attention to clearly communicating your assumptions, thoughts, and problem solving processes, and to justifying your procedures and conclusions. This is an opportunity to show off what you have learned and how you can apply that knowledge.

Sign one of the following two statements.

Statement: I worked this test in compliance with the above directions.

_____(Signature)

Statement: I am unable to sign the statement above due to the exception(s) listed below.

_____(Signature)

Part II. Problems (10 points each) Start each problem on a fresh page. Place your name at the top of each page. Be sure to carefully justify your reasoning. Clearly state your conclusions using complete sentences.

1. Employ exemplary proof technique and use of language in proving the following theorem.

    The transformation matrix D for a direct dilation with center (h, k, 1) and ratio r is

    $$D = \begin{bmatrix} r & 0 & h(1-r) \\ 0 & r & k(1-r) \\ 0 & 0 & 1 \end{bmatrix}.$$

2. Three transformations are to be performed in the following order:

    Translate through (3, -1)
    Reflect through the line y = x
    Rotate through a counterclockwise angle of 45°

    a. Suppose T is the single affine transformation that is the composition of the three named above. Show how to use matrix multiplication to determine the transformation matrix for T. Express your answer using exact numerical values.

    b. Show how to determine $T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right)$. Use exact numerical values.

3. Consider the strain S with axis [-1 2 0] and ratio 3.

    a. Show how to determine a matrix representation for S.

    b. Show how to use the GSP to display the image of the unit square under S.

4. Show how to determine all invariant points and lines under the transformation T whose matrix representation is shown below.

    $$T = \begin{bmatrix} \left(\dfrac{-5}{13}\right) & \left(\dfrac{27}{13}\right) & 0 \\ \left(\dfrac{-12}{13}\right) & \left(\dfrac{31}{13}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$