Some Transformations on E^2

Every translation can be expressed as the composition of two reflections where the reflections are in parallel lines; every rotation can be expressed as the composition of reflections where the reflections are in concurrent lines.

Every isometry can be expressed as the composition of at most three reflections.

A one-to-one transformation $T:E^2 \xrightarrow{onto} E^2$ is a *dilation* with center O with nonzero ratio r iff $T(O) = O$ and for each P, Q $\in E^2$, $d(T(P),T(Q)) = rd(P,Q)$.

A one-to-one transformation $T:E^2 \xrightarrow{onto} E^2$ is a *shear* if all points P on a given line L are invariant while all other points are shifted parallel to L by a distance proportional to their perpendicular distance from L.

An *affine transformation* is any transformation that preserves collinearity (i.e., all points lying on a line initially still lie on a line after transformation) and ratios of distances (e.g., the midpoint of a line segment remains the midpoint after transformation). An affine transformation is also called an *affinity*. Affine transformations also preserve parallelism.

In general, an affine transformation is a composition of rotations, translations, dilations, and shears.

The general matrix for affine transformations is

$$T = \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix}.$$

A *glide reflection* is the composition of a reflection and a translation in a direction parallel to the line of reflection.

A one-to-one transformation $T:E^2 \xrightarrow{onto} E^2$ is a *similitude* with nonzero ratio r iff for each P, Q $\in E^2$, $d(T(P),T(Q)) = rd(P,Q)$.

If a transformation preserves perpendicularity it is called an *orthomap*.

Some Matrix Representations

Orthomap: $\begin{bmatrix} a & -\delta b & 0 \\ b & \delta a & 0 \\ 0 & 0 & 1 \end{bmatrix}, \delta = \pm 1.$

Similitude: $\begin{bmatrix} a & -\delta b & e \\ b & \delta a & f \\ 0 & 0 & 1 \end{bmatrix}, (\delta = \pm 1).$

Dilation with center O(h,k) and ratio r: $\begin{bmatrix} r & 0 & e \\ 0 & r & f \\ 0 & 0 & 1 \end{bmatrix}$, *where e = h-rh and f = k – ak.*

Isomerty: $\begin{bmatrix} a & -\delta b & e \\ b & \delta a & f \\ 0 & 0 & 1 \end{bmatrix}, (\delta = \pm 1)$ *and* $a^2 + b^2 = 1$.

Rotation about O(0,0): $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Reflection in Line (ax + by = c): $\frac{1}{a^2+b^2}\begin{bmatrix} b^2-a^2 & -2ab & 2ac \\ -2ab & a^2-b^2 & 2bc \\ 0 & 0 & 1 \end{bmatrix}.$

Strain with ratio r and axis [0 1 0]: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

Shear with axis [0 1 0] and ratio r: $\begin{bmatrix} 1 & r & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$