MATH 406 Supplementary Notes for Section 3.2

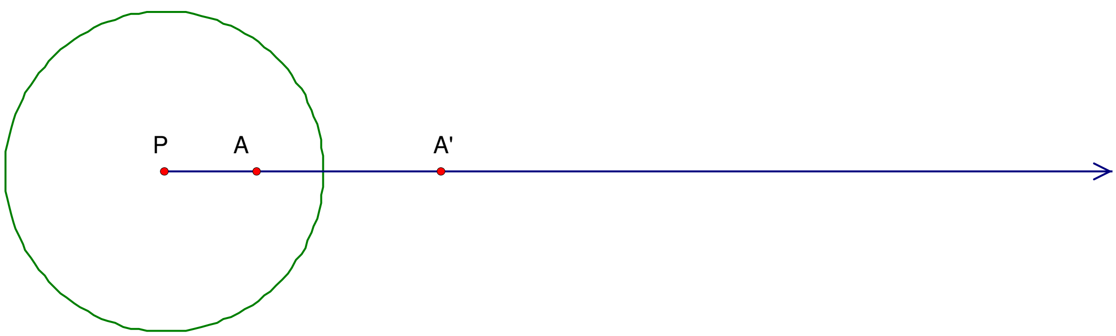
Let E denote the Euclidean plane and $\text{P} \in E$. The set $E - \{\text{P}\}$ is called a *punctured plane.*

Definition of Circular Inversion

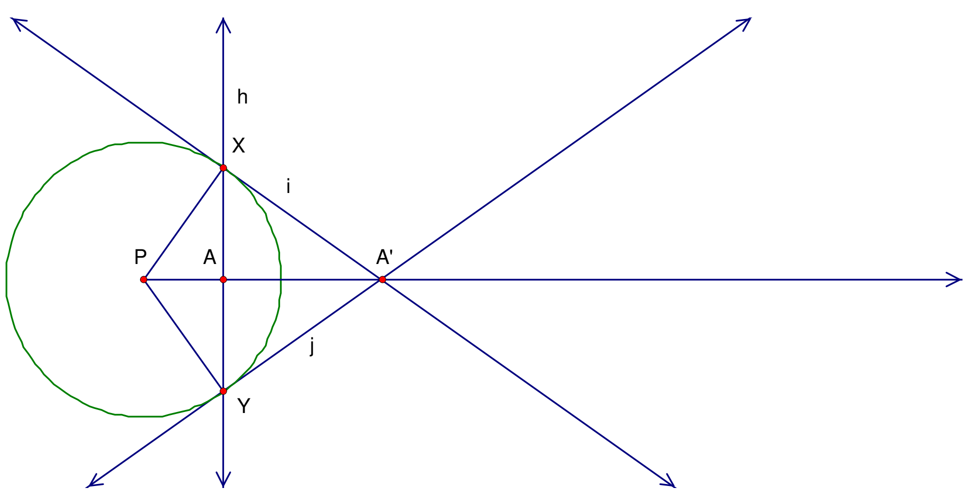
Let C be a circle with center at P and radius r. The ***inversion of E – {P} in the circle C*** is a mapping

$$f : E - \{\text{P}\} \rightarrow E - \{\text{P}\}$$

that associates each point $\text{A} \in E - \{\text{P}\}$ with a point $\text{A'} \in E - \{\text{P}\}$ such that A' lies on the ray OA and $\text{OA} \cdot \text{OA'} = \text{r}^2$.



Constructing Inverse Points for Points A Inside Circle C



Construct ray PA.
Construct line h perpendicular to ray AP at A.
Label intersections of line h with circle C X and Y respectively.
Construct segments PX and PX.
Construct line i perpendicular to segment PX at X.
Construct line j perpendicular to segment PY at Y.
The intersection of lines i and j is the inverse point A'.

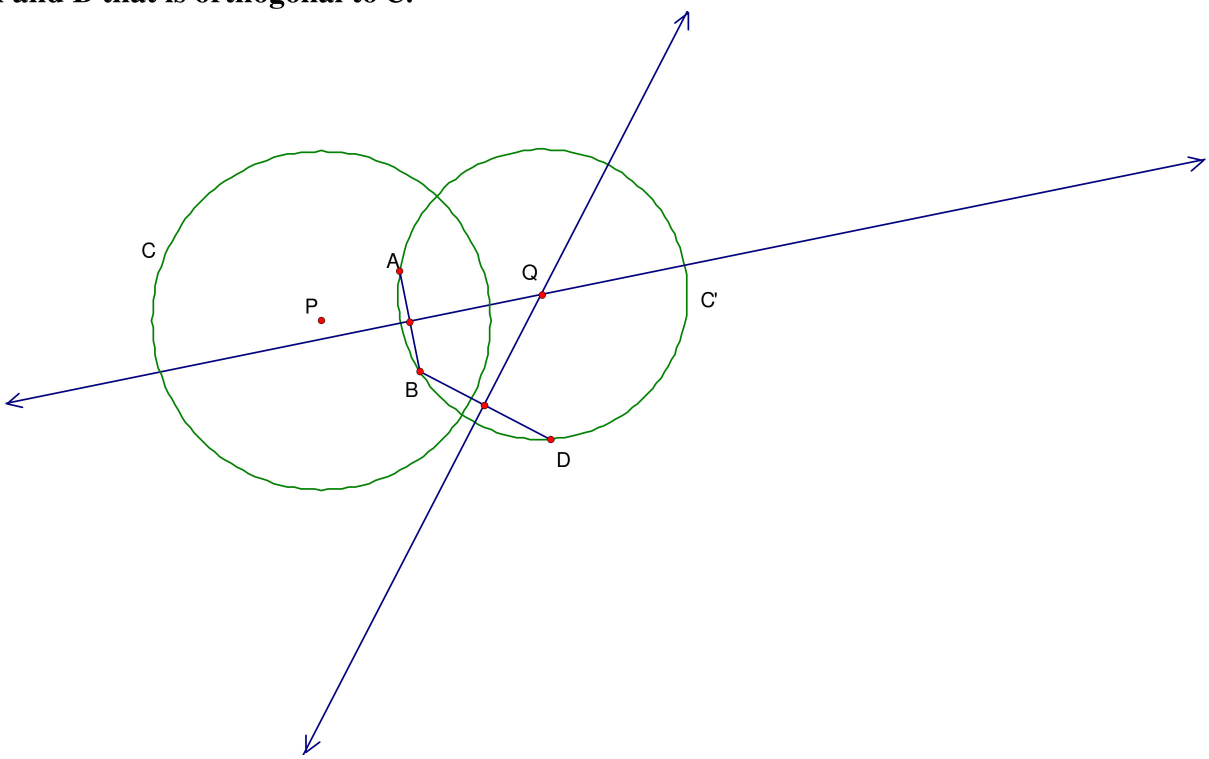
Optional Exercise:

Prove that the construction outlined above does indeed produce A' as claimed.

Optional Exercise:

Explain, illustrate and prove, how to construct inverse points for points A outside of Circle C.

Given a circle *C* and points A and B inside *C,* construct a circle *C'* containing points A and B that is orthogonal to *C.*



Construct point D so that D is the image of B under inversion in the circle C.
Construct the circle C' passing through the points A, B, and C.
C' is orthogonal to C and C' contains A and B.