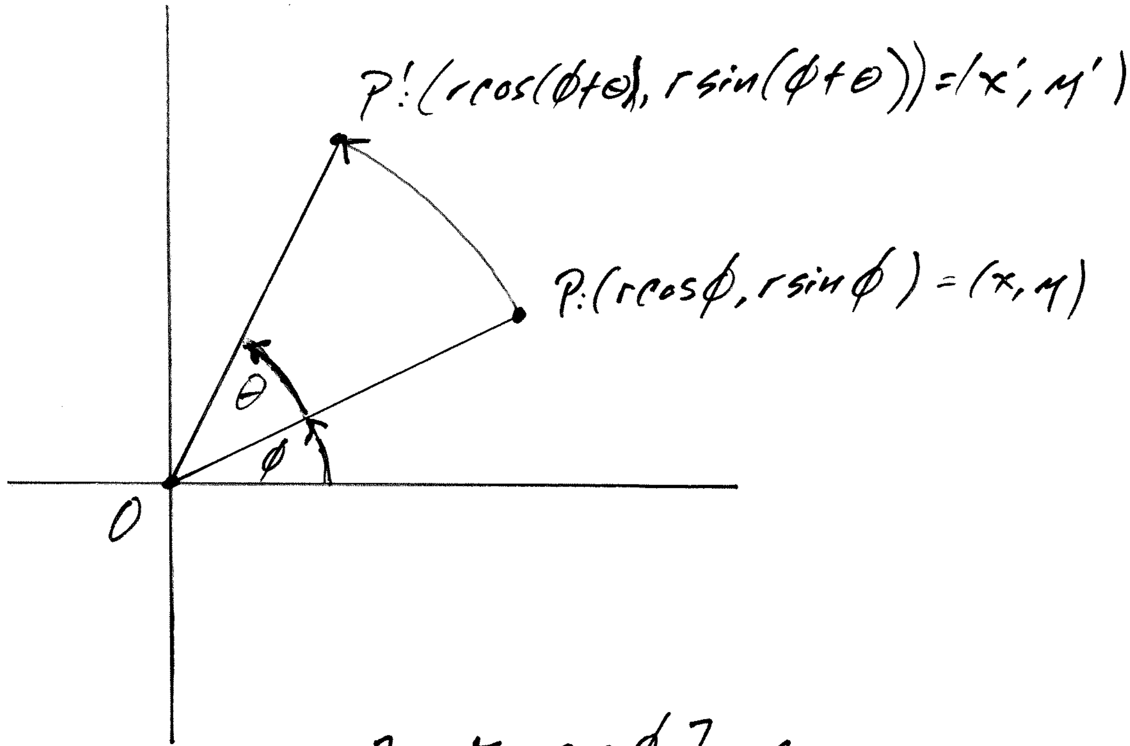
Consider rotation of $P(x,y)$ through angle θ about O.



Suppose $P = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} r\cos\phi \\ r\sin\phi \\ 1 \end{bmatrix}$ for some r, ϕ.

$$P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} r\cos(\phi+\theta) \\ r\sin(\phi+\theta) \\ 1 \end{bmatrix} = \begin{bmatrix} r(\cos\phi\cos\theta - \sin\phi\sin\theta) \\ r(\sin\phi\cos\theta + \cos\phi\sin\theta) \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (r\cos\phi)\cos\theta - (r\sin\phi)\sin\theta \\ (r\sin\phi)\cos\theta + (r\cos\phi)\sin\theta \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ y\cos\theta + x\sin\theta \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ 1 \end{bmatrix}$$

So $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

Exercise: A square with vertices $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ is rotated through $\frac{\pi}{4}$ about $O\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Find its image.