Some Notes for Sections 4.2 & 4.3

The **Euclidean plane**, denoted by E^2, is {(x,y,1): x, y ∈ R}. We will denote elements of E^2 by column vectors $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$.

A transformation T: $E^2 \rightarrow E^2$ is a **linear transformation of $\mathbf{E^2}$** iff $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for each **u**, **v** ∈ E^2 and $T(\lambda\mathbf{u}) = \lambda T(\mathbf{u})$ for each λ ∈ R and **u** ∈ E^2.

T: $E^2 \rightarrow E^2$ is an **invertible transformation of $\mathbf{E^2}$** iff it is 1-1 and onto E^2.

Many transformations T: $E^2 \rightarrow E^2$ can be defined by a rule of the form $T(\mathbf{u}) = A\mathbf{u}$ where A is a 3 *x* 3 matrix of the form

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \text{ where } a_{ij} \in R.$$

A is called the **matrix representation** for T. For some matrices A, T is a linear transformation.

So, if $\mathbf{u} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$, then $T(\mathbf{u}) = A\mathbf{u} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$.

Write matrix representations for each of the following transformations of E^2.

Translation (slide) defined by (e,f)

Counter clockwise rotation through θ about the origin

Reflection in the y-axis

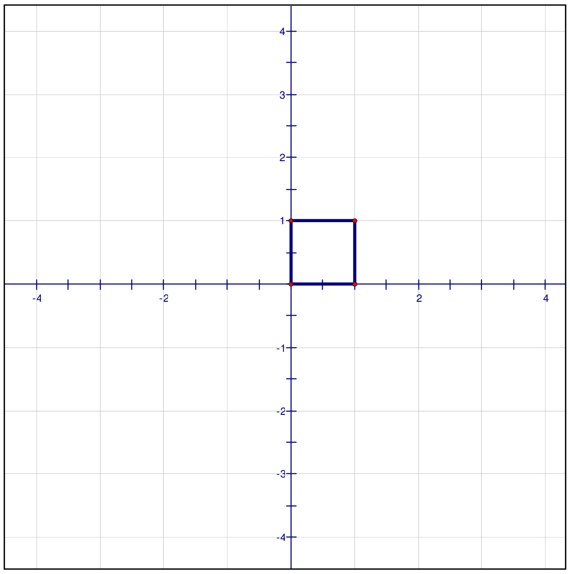
Reflection in the x-axis

Reflection in the line y = x

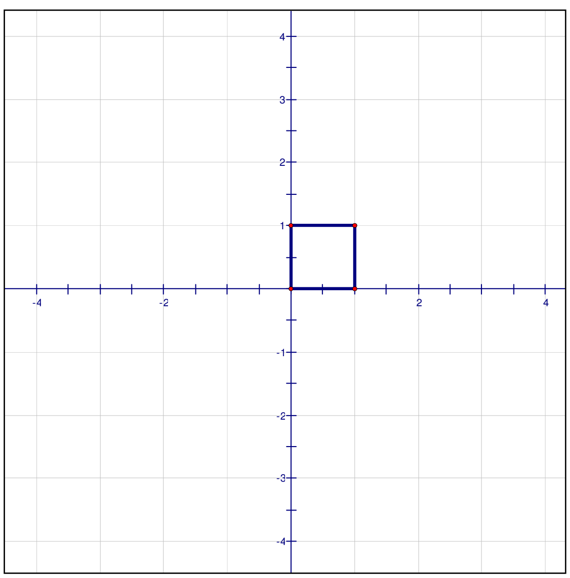
Reflection in the line y = -x

Use both geometric thinking and matrix methods to find the image of the unit square under each of the following transformations.

T_1: a counter clockwise rotation of 60^0 about the origin.



T_2: a translation through (2,1) followed by a reflection through the line y = -x.



Specify the inverse of each of the transformations T_1 and T_2.