MATH 406 Session #33

Find all invariant lines under the transformation $T_2\left(\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$.

T_2 is a reflection in the line $\begin{bmatrix} -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$ **; so** $T_2^{-1}\left(\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$.

Applying Theorem 4.2.4 to this situation, we seek $\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = k\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$ **for an arbitrary non-zero constant** k**.**

$\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$ **if** $\begin{bmatrix} u_2 & u_1 & u_3 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$.

That is if $u_2 = u_1$ **and** $u_1 = u_2$ **and** $u_3 = u_3$.

Hence any line with equation of the form $k\begin{bmatrix} 1 & 1 & l \end{bmatrix}\begin{bmatrix} y \\ y \\ 1 \end{bmatrix} = 0$ **where** k **is an arbitrary non-zero constant and** l **is an arbitrary constant is invariant under** T_2**. Those are simply the lines with equations, in standard form, of the form** k**x +** k**y +** kl **= 0 where** k **and** l **are arbitrary constants and** k **is not zero. These are all lines perpendicular to the line y = x. In our previous exercise, we showed that points on the line y = x are invariant.**

So, the line y = x, or $\begin{bmatrix} -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$**, is also invariant under** T_2**.**

Consequently, the lines y = x and all those of the form y = -x + c for some constant c are invariant under T_2**. Alternatively, the lines**

$\begin{bmatrix} -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$ **and** $\begin{bmatrix} 1 & 1 & c \end{bmatrix}\begin{bmatrix} y \\ y \\ 1 \end{bmatrix} = 0$ **for any constant** c **are invariant under** T_2**.**